MATHEMATICS
3º ESO

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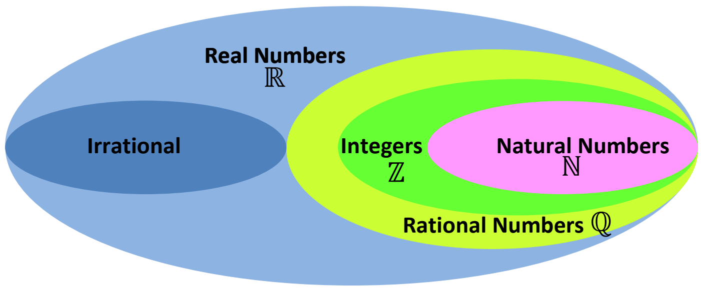
LESSON 1: FRACTIONS AND DECIMALS

Keywords

Set Element Natural FRACTION Simplify Equivalent NUMERATOR Denominator Compare Period Integer Cross-Multiplying Simplest form PERCENTAGES Recurring/Repeating Decimal Terminating Decimal increase decrease Compound Interest Principal Interest rate Borrow Invest

1. NUMBERS

So far, you have studied different sets of numbers: natural numbers, integers, fractions or rational numbers and real numbers.



Remember: A ***set*** is a collection of objects, these objects are called ***elements***.

- *Natural (or counting) Numbers*: 1, 2, 3, 4 …
- *Whole numbers*: 0, 1, 2, 3, 4 …
- *Integers*: …, -3, -2, -1, 0, 1, 2, 3, …
- *Rational numbers*: Natural numbers, integers and fractions are rational numbers. A fraction is formed by two numbers as

$$\frac{5}{3} \begin{array}{l}\leftarrow \text{Numerator}\\ \leftarrow \text{Denominator}\end{array}$$

In this lesson you will study rational numbers.

2. EQUIVALENT FRACTIONS

Equivalent fractions are those that represent the same value. If two fractions are equivalent we write "equals": $\frac{9}{12} = \frac{3}{4}$

You can check if two fractions are equivalent by ***cross-multiplying.***

Example:

$$\frac{2}{4} \times \frac{3}{6}$$

$$6 \cdot 2 = 4 \cdot 3$$

$$12 = 12$$

LESSON 1

2.1 Simplifying fractions

Sometimes you can divide the top and bottom of a fraction by the same number. This is called ***cancelling down***. It is also called ***simplifying*** the fraction. You can express a fraction in its ***simplest form*** dividing the numerator and the denominator by the ***highest common factor*** **(HCF)**.

Example: Simplify $\frac{180}{150}$

HCF (180, 150) = 30 $\Rightarrow$ $\frac{180}{150} = \frac{6}{5}$ (They are equivalent fractions) (:30 top and bottom)

2.2. Comparing fractions

When two fractions have the same denominator we compare them just comparing the numerators.

Example: $\frac{4}{15} < \frac{34}{15}$

If the fractions don't have the same denominator you need to find the lowest common denominator and then you can compare them.

Example: $\frac{3}{6}$ and $\frac{5}{4}$

1. We calculate the LCM of 6 and 4.
   LCM (6, 4)= 12

   so, $\frac{3}{6} = \frac{?}{12}$ and $\frac{5}{4} = \frac{?}{12}$

2. We calculate the new numerators.
   12: 6 = 2 and $2 \cdot 3 = 6 \Rightarrow$ $\boxed{\frac{3}{6} = \frac{6}{12}}$

   12: 4 = 3 and $3 \cdot 5 = 15 \Rightarrow$ $\frac{?}{12} = \frac{?}{12}$

Now the fractions have the same denominator and we can compare them:

$$\frac{6}{12} < \frac{15}{12} \Rightarrow \frac{3}{6} < \frac{5}{4}$$

3. OPERATIONS WITH FRACTIONS

- To *add* and *subtract* fractions they must have the ***same denominator.***
- To *multiply* fractions:

Example:

$$\frac{2}{4} \cdot \frac{3}{5} = \frac{2 \cdot 3}{4 \cdot 5} = \frac{6}{20} \overset{\text{Simplify}}{=} \frac{3}{10}$$

➢ To *divide* fractions you use cross-multiplication.

Example:

$$\frac{2}{4} : \frac{3}{5} = \frac{2 \cdot 5}{4 \cdot 3} = \frac{10}{12} \overset{\text{Simplify}}{=} \frac{5}{6}$$

➢ Finally, when there are *different operations* you must remember **BODMAS**.

Example:

$$\frac{1+\frac{2}{3}}{\frac{1}{2}-\frac{4}{3}:\frac{2}{5}} = \frac{\frac{3}{3}+\frac{2}{3}}{\frac{1}{2}-\frac{20}{6}} = \frac{\frac{5}{3}}{\frac{1}{2}-\frac{10}{3}} = \frac{\frac{5}{3}}{\frac{3}{6}-\frac{20}{6}} = \frac{\frac{5}{3}}{\frac{-17}{6}} = \frac{5}{3} : \frac{-17}{6} = \frac{30}{-51} = -\frac{10}{17}$$

4. FRACTION OF A QUANTITY

You know that in Maths "of" means "times". You can find three different types of exercises about fractions of quantities. Here you can see examples.

Example 1: There are 35 students in a classroom, $\frac{2}{5}$ of them are ill, how many students are ill?

Solution: You have to calculate $\frac{2}{5}$ of 35= $\frac{2}{5} \cdot 35 = \frac{2 \cdot 35}{5} = 14$ students are ill.

Example 2: In a class there are 14 students who are ill, they represent $\frac{2}{5}$ of the total number of students, how many students are there in the classroom?

Solution: Now, you don't know the total number of students, it is x, so $\frac{2}{5}$ of $x=14 \Rightarrow \frac{2}{5} \cdot x=14 \Rightarrow x=\frac{14 \cdot 5}{2}=35$ students in the classroom.

Example 3: 14 students, of a class of 35, are ill, what fraction of students are ill?

Solution: Now, you don't know the fraction, but it is very easy to find it, you write in the numerator the number of ill students and in the denominator the total number of students. Finally you must simplify the fraction.

$$\frac{14}{35} = \frac{2}{5}$$

5. DECIMAL NUMBERS

Firstly, you need to remember how to classify decimal numbers.

DECIMAL NUMBERS
- Exact or terminating: 0,247
- Recurring or repeating: 12,35353535…= $12,\widehat{35}$
- Other decimals: π=3,141592…

Remember: In Spanish we distinguish two types of recurring decimals, they are "periódicos …………." and "periódicos …………".

LESSON 1

Exercise: Write another example of "other decimals".

Some decimal numbers can be expressed as fractions, they are the terminating and the recurring decimals.

5.1. How to convert a fraction into a decimal

You just divide the numerator by the denominator.
Example: $\frac{2}{3}=2:3=0{,}66666\ldots=0{,}\widehat{6}$

5.2. How to convert a decimal into a fraction

We need to distinguish if the decimal number is terminating or recurring, otherwise, it's not possible to write it as a fraction.

- If it is a terminating decimal: you write in the numerator the number without comma and in the denominator 1 and as many zeros as the number of decimal digits. Finally, simplify the fraction.

- If it is a recurring decimal there are two different methods, you studied the first of them last year.
  - Method 1: Memorizing a formula.

    Example 1: $3{,}\widehat{12}=\frac{312-3}{99}=\frac{309}{99}=\frac{103}{33}$

    Example 2: $3{,}4\widehat{12}=\frac{3412-34}{990}=\frac{3378}{990}=\frac{563}{165}$

    $$A,B\widehat{C}=\frac{ABC-AB}{9\ldots90\ldots0}$$

    Number of 9's= Number of digits in C
    Number of 0's= Number of digits in B

  - Method 2: Multiplying by 10, 100, 1000..., subtracting and using algebra.

    Example 1:

    $x=3{,}\widehat{12}=3{,}12121212\ldots$ next, you multiply by 100.

    $100x=312{,}\widehat{12}1212\ldots$, if you subtract

    $$\begin{array}{rl} 100x= & 312{,}121212\ldots \\ -x= & 3{,}121212\ldots \\ \hline 99x=309 & \Rightarrow\ x=\frac{309}{99}=\frac{103}{33} \end{array}$$

    Finally, $3{,}\widehat{12}=\frac{103}{33}$.

    Example 2:

    $x=3{,}4\widehat{12}=3{,}412121212\ldots$ first you multiply by 10, then all the decimal digits are repeated.

    $10x=34{,}121212\ldots$, next, you multiply by 1000 (to move the decimal comma two more places), $1000x=3412{,}121212\ldots$ and then subtract

    $$\begin{array}{rl} 1000x= & 3412{,}121212\ldots \\ -10x= & 34{,}121212\ldots \\ \hline 990x=3378 & \Rightarrow\ x=\frac{3378}{990}=\frac{563}{165} \end{array}$$

    Finally, $3{,}4\widehat{12}=\frac{563}{165}$.

6. PERCENTAGES

As you already know about percentages, we will revise this topic solving easy exercises.

Exercise 1: A school has 1248 students, and 48% of them are girls. How many girls are there in the school?

Exercise 2: 25 % of the teachers in a school teach Maths. If there are 50 Maths teachers, how many teachers are there in the school?

Exercise 3: 24 students in a class took an algebra test. If 18 students passed the test, what percent did not pass?

Exercise 4: (percentage increase) Julia earns a salary of £47800 per year. She gets a 2,7% pay rise. Calculate her new salary.

Exercise 5: (percentage decrease) Paul got a discount of 40% in a CD that cost 18€, how much did he finally pay?

Exercise 6: Jimmy got a raise from $6,00 an hour to $8,00 an hour. This was a raise of what percent?

Exercise 7: A pair of jeans that yesterday cost 30€, today has a special discount of 50%, the shop assistant told me that tomorrow the price will increase 50%, how much will the pair of jeans cost tomorrow?

7. COMPOUND INTEREST

You earn ***interest*** when you ***invest*** money in a savings account in a bank. However, you pay interest if you ***borrow*** money from a bank.

The original sum you invest or borrow is called the ***principal*** (P) and the per cent is called the ***interest rate*** (r).

Interest can be:
- Simple interest: it is not added to the principal.
- Compound interest: added to the principal to earn more interest.

You will understand the difference with this example.

Example:
Calculate the interest when 1000€ are invested for 4 years at

a) 5% simple interest (SI)
For 1 year, SI= 5% ·1000 = 50 €
For 4 years, SI= 4 · 50 = 200€
Total= principal + interest=
=1000 + 200 = 1200€

b) 5% compound interest (CI)
1st year's CI= 5% · 1000= 50 €
New principal= 1000 +50 = 1050€
2nd year's CI= 5% · 1050= 52,50 €
New principal=1050 +52,50 = 1102,50€
3rd year's CI= 5% · 1102,50= 55,13 €
New principal= 1102,50 +55,13 = 1157,63€
4th year's CI= 5% · 1157,63= 57,88 €
New principal= 1157,63 +57,88 = 1215,51€

Formula: New principal= $P\cdot\left(1+\frac{r}{100}\right)^n$
n=numbers of years

LESSON 1

PRONUNCIATION

- Compare | kəmˈpeə |
- Compound Interest | kəmˈpaʊnd ˈɪntrəst |
- Cross-Multiplying| krɒs ˈmʌltɪplaɪɪŋ |
- Denominator | dɪˈnɒmɪneɪtə |
- Element | ˈelɪmənt |
- Equivalent | ɪˈkwɪvələnt |
- Fraction | ˈfrækʃən |
- Integer| ˈɪntɪdʒə |
- Natural| ˈnætʃrəl |
- Numerator| ˈnjuːməreɪtə |
- Percentages | pəˈsentɪdʒɪz |
- Period | ˈpɪərɪəd |
- Recurring/Repeating Decimal | rɪˈkɜːrɪŋ rɪˈpiːtɪŋ ˈdesɪməl|
- Set | set |
- Simplest form | ˈsɪmplɪst fɔːm |
- Simplify | ˈsɪmplɪfaɪ |
- Terminating Decimal | ˈtɜːmɪneɪtɪŋ ˈdesɪməl |
- To decrease | tu dɪˈkriːs |
- To increase | tə ɪnˈkriːs |
- Principal |ˈprɪnsəpəl|
- Interest rate |ˈɪntrəst reɪt|
- Borrow |ˈbɒrəʊ|
- Invest |ɪnˈvest |

WORKSHEET

1. Order from lowest to greatest:

   $\frac{3}{5}$; $\frac{6}{9}$; $\frac{17}{15}$; $\frac{11}{3}$

2. Copy and complete these sets of equivalent fractions:

   a) $\frac{3}{4} = \frac{\;}{8} = \frac{\;}{12} = \frac{\;}{16} = \frac{\;}{24}$

   b) $\frac{2}{7} = \frac{\;}{14} = \frac{\;}{21} = \frac{\;}{28} = \frac{\;}{42}$

3. Calculate:

   a) $\dfrac{\left(\frac{5}{3}+\frac{4}{7}\right):\frac{3}{2}}{\frac{3}{4}-\frac{1}{5}\cdot\frac{6}{7}}$

   b) $\dfrac{3}{1-\dfrac{3}{1-\frac{3}{4}}}$

4. Write as fractions in their simplest form:

   a) 0,48

   b) $0,\widehat{27}$

   c) $0,41\widehat{6}$

5. Kim made two pies that were exactly the same size. The first pie was a cherry pie, which she cut into 6 equal slices. The second was a pumpkin pie, which she cut into 12 equal pieces. Kim takes her pies to a party. People eat 3 slices of cherry pie and 6 slices of pumpkin pie. Did people eat more cherry pie or pumpkin pie?

6. Gail wants to work out what her weight will be if it increases by 4%. What should she multiply her present weight by?

7. John buys an old car for £8400. He spends £1040 on repairs, then sells the car on eBay for £14900. Find

   a) His actual profit.

   b) His percentage profit to the nearest 1%.

8. Find the simple interest on 7000€ invested for 4 years at 6% per year.

9. Patricia earns 100€ a year in interest from her savings, which are invested at 8% simple interest per year. Calculate how much she has in her savings.

10. Which of these options earns the most interest when a principal of £5000 is invested for

    a) 8 years at 6% simple interest

    b) 6 years at 8% compound interest?

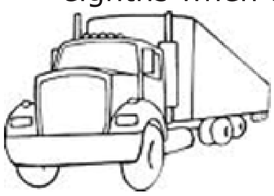
11. £2000 are invested at 6,5% compound interest. Find the principal after 15 years.

LESSON 1

WORD PROBLEMS

1. A bag of flour weighs 2,25 kg. More flour is added and the weight of the bag of flour is increased by three fifths. What is the new weight of the bag of flour?

2. A loaded lorry has a total weight of 13,2 tonnes. This weight is decreased by five eighths when the load is removed. Find the weight of the lorry without the load.



3. Last year 204 cars were imported by a garage. This year the number of cars imported has increased by five twelfths. How many cars have been imported this year?

4. There are 225 houses on a state. Of these houses, 85 have no garage. What fraction of houses have no garage?

5. A newspaper has 14 columns of photographs and 18 columns of advertisements. What fraction of the paper is advertisements?

6. Find the difference between $\frac{3}{5}$ of 36 miles and $\frac{2}{3}$ of 30 miles.

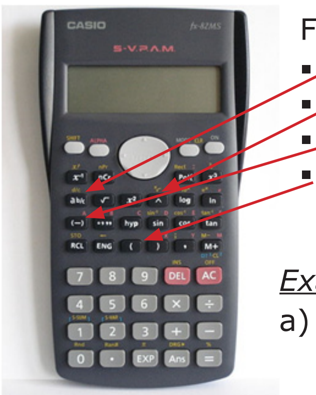
CALCULATOR METHODS

LESSON 1

Use your calculator effectively and efficiently. It is important to know when you should and you should not use it.

You need to know how to enter calculations into a calculator, and how to interpret the calculator ***display.***

This unit will show you how to use your calculator to make calculations with fractions and powers.



Find these keys in your calculator:

- [a b/c]
- [^] or [y^x]
- [(−)] or [+/−]
- [( ] [ )]

Examples: Calculate and simplify:

a) $\frac{2}{3} + \frac{5}{4}$

Press these keys: 2 [a b/c] 3 [+] 5 [a b/c] 4 [=]. In the display you can see something like 1⌟11⌟12 which represents $1+\frac{11}{12}$, pressing [SHIFT] [a b/c] you get the fraction $\frac{23}{12}$ in its simplest form.

b) $$\frac{\frac{3}{5}+\frac{1}{9}}{\frac{2}{3}+\frac{3}{10}}$$

Press these keys:

[( ] 3 [a b/c] 5 [+] 1 [a b/c] 9 [ )] [÷] [( ] 2 [a b/c] 3 [+] 3 [a b/c] 10 [ )]. In the display you can see the simplest form $\frac{64}{87}$.

c) $(-2)^4$

Press these keys: [( ] [(−)] 2 [ )] [^] or [y^x] 4.

Exercise: Calculate using your calculator.

a) $$\frac{\frac{2}{5}-\frac{1}{7}}{\frac{1}{6}+\frac{2}{9}}$$

b) $(-3)^{11}$

c) $(-5)^3 \cdot 7^4$

d) $$\frac{\left(\frac{7}{6}-\frac{11}{3}\right)\cdot\frac{2}{5}-\frac{1}{7}}{2+\frac{3}{10}}$$

LESSON 2

LESSON 2: POWERS AND ROOTS. APPROXIMATE NUMBERS

Keywords

Power Root Exponent BASE Radical **Radicand** INDEX
Like terms Accurate **Inaccurate** Significant digits
Absolute Relative SCIENTIFIC NOTATION

1. POWERS WITH POSITIVE EXPONENT

$2^5=2\cdot2\cdot2\cdot2\cdot2$ Two ***to the power of*** 5 or two ***to the fifth***

Base ⟶ 2^5 ⟵ Exponent

Special powers:

- 5^2 five ***squared***
- 4^3 four ***cubed*** or four to the third

Properties of powers:

1. $a^0 = 1$
2. $a^1 = a$

3. The power of a product is the product of the powers.

$$(a \cdot b)^n = a^n \cdot b^n$$

Example: $(2 \cdot 3)^4 = 2^4 \cdot 3^4$

4. The power of a quotient is the quotient of the powers.

$$(a : b)^n = a^n : b^n$$

Example: $(2 : 3)^4 = 2^4 : 3^4$

5. When multiplying powers *of the same base*, you keep the same base and add the exponents.

$$a^n \cdot a^m = a^{n+m}$$

Example: $5^3 \cdot 5^4 = 5^7$

6. When dividing powers of the same base, you keep the same base and subtract the exponents.

$$a^n : a^m = a^{n-m}$$

Example: $7^8 : 7^5 = 7^3$

7. When powering a power, you keep the base and multiply the exponents.

$$(a^n)^m = a^{n \cdot m}$$

Example: $(4^3)^2 = 4^6$

8. When the base is a negative number and the exponent is even, the result is positive. If the exponent is odd, the result is negative.

Examples: $(-2)^3=-8$ $(-3)^2=9$

2. POWERS WITH NEGATIVE EXPONENT

We usually use powers with positive exponent but, when you find a power with a negative exponent you can convert it into a fraction with a positive one.

$$a^{-n} = \frac{1}{a^n}$$
$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Examples:

a) $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$

b) $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = 4^2$

c) $(-5)^{-4} = \left(-\frac{1}{5}\right)^4$

Note: The properties of powers with positive exponent are true for powers with negative exponent too.

3. ROOTS

3.1. Square roots

$$\sqrt{a} = b \longrightarrow b^2 = a$$

a ***radicand***

b ***root***

The opposite of square a number is calculating its square root.

Example: $\sqrt{9}=3$ The square root of nine equals 3 because $3^2=9$

3.2. Cube roots

$$\sqrt[3]{a}=b \longrightarrow b^3 = a$$

The opposite of cube a number is calculating its cube root.

Example: $\sqrt[3]{8}=2$ The cube root of 8 equals 2 because $2^3=8$

3.3. Other roots

$$\sqrt[n]{a}=b \longrightarrow b^n = a$$

The opposite of cube a number is calculating its cube root.

Example: $\sqrt[4]{625}=5$ The fourth root of 625 equals 5 because $5^4=625$

4. RADICALS

A ***radical*** is an expression that has a root.
Every radical can be expressed as a power with a rational exponent:

$$\sqrt[n]{a^m}=a^{\frac{m}{n}}$$

Examples: a) $\sqrt{5}=5^{\frac{1}{2}}$ b) $\sqrt[3]{7}=7^{\frac{1}{3}}$