

MATHEMATICS 3 ESO

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educàlia
editorial

Versión revisada 2023

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Imprime: Grupo Digital 82, S.L.

ISBN: 978-84-943962-5-0

Depósito Legal: V -2374 -2015

Printed in Spain/Impreso en España.

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"Thank you, Sue, I am privileged to have you as a friend.

*I would also like to thank my dear students (2013-2016)
for encouraging me to write this book".*

PREFACIO

“La esencia de las Matemáticas no consiste en complicar lo que es simple, sino en simplificar lo que es complicado”.

Stanley Gudder

La enseñanza de las Matemáticas en la Educación Secundaria, lo sabemos bien los que nos dedicamos a ello, puede llegar a ser una tarea difícil y más aún si se realiza en una lengua que no es la lengua materna del alumnado.

Este libro surge precisamente de mi interés por elaborar un material sencillo y atractivo para el estudiante y que al mismo tiempo sea fiel al currículo de esta etapa. El hecho es que facilita muchísimo la labor del profesorado pues se trata de un compendio de contenidos, actividades y problemas que puede ser utilizado perfectamente como libro de texto, lo que es una gran ventaja como he podido comprobar personalmente en mi labor como profesora.

El libro está estructurado en unidades didácticas. Cada una de ellas consta de:

- Índice detallado.
- “Key Vocabulary”, es decir, el **vocabulario técnico de Matemáticas en inglés** que necesita el alumnado específicamente para cada unidad.
- Esquema introductorio, que informa de lo que se va a estudiar en la unidad correspondiente.
- **Contenidos**, que es la parte esencial de la misma y que está plagada de ejemplos, casos particulares, diagramas, gráficos... todo ello elaborado cuidando el detalle y el color.
- Una **tabla** donde se recoge el vocabulario nuevo aprendido y que el alumno debe completar con la fonética de cada palabra y su significado.
- Y por último se incluyen una **amplia colección de actividades, ejercicios y problemas de cada unidad didáctica**.

Se trata de un material adaptable a diversas metodologías pues está elaborado teniendo en cuenta los principios de la metodología AICLE (CLIL) que ofrece la posibilidad de aprender los contenidos curriculares de la asignatura de Matemáticas a la vez que permite practicar la lengua inglesa aprendida en etapas previas incrementando su bagaje de vocabulario técnico específico de Matemáticas en Inglés.

El hecho de que esté íntegramente elaborado en inglés constituye una ventaja para la inmersión total del alumnado en la lengua inglesa.

El alumno asimila palabras, frases y vocabulario cotidiano de la lengua inglesa además de estructuras y vocabulario específico de la propia asignatura de Matemáticas.

Es más, como es el propio alumno el que completa la tabla fonética de cada unidad, va interiorizándola sin apenas darse cuenta; tabla que puede ampliar con los términos que cada uno decida individualmente. Se trata en definitiva de un material que el alumno mismo ayuda a elaborar según sus necesidades particulares y que al final del año escolar le habrá servido para confeccionar su propia lista de vocabulario específico.

Otra ventaja de estos materiales es que por la **sencillez y concreción** con que están definidos los conceptos en el libro, se facilita el aprendizaje y se produce un impacto en la conceptualización, es decir, el alumno llega a ser capaz de pensar directamente en lo que se dice aunque esté expresado en otra lengua centrándose en los contenidos curriculares de la materia. Este aspecto ayuda a ampliar su mapa conceptual del pensamiento y a desarrollar en mayor medida sus competencias.

Y si alguna cosa más hubiera que destacar, personalmente destacaría la **motivación** que se logra en el alumnado al trabajar con este material. Por su estructura práctica, el colorido elegido al detalle para hacerlo atractivo, las listas de recursos web que se facilitan, porque las actividades y problemas planteados se ajustan a los contenidos y al nivel con propiedad, o por todo ello unido, se produce un efecto participativo y motivador que de otra forma es difícil alcanzar en la actualidad.

Por último decir que este libro está escrito con la ilusión de compartir y transmitir dos de mis grandes pasiones, la lengua inglesa y las Matemáticas, de la manera más sencilla posible.

ROSARIO CARRASCO TORRES

MATHEMATICS

3 ESO

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REMEMBER THAT:

	CARDINAL	ORDINAL		CARDINAL	ORDINAL
1	One	First (1 st)	21	Twenty-one	Twenty-first (21 st)
2	Two	Second (2 nd)	22	Twenty-two	Twenty-second (22 nd)
3	Three	Third (3 rd)	23	Twenty-three	Twenty-third (23 rd)
4	Four	Fourth (4 th)	24	Twenty-four	Twenty-fourth (24 th)
5	Five	Fifth (5 th)	25	Twenty-five	Twenty-fifth (25 th)
6	Six	Sixth (6 th)	26	Twenty-six	Twenty-sixth (26 th)
7	Seven	Seventh (7 th)	27	Twenty-seven	Twenty-seventh (27 th)
8	Eight	Eighth (8 th)	30	Thirty	Thirtieth (30 th)
9	Nine	Ninth (9 th)	40	Forty	Fortieth (40 th)
10	Ten	Tenth (10 th)	50	Fifty	Fiftieth (50 th)
11	Eleven	Eleventh (11 th)	100	One hundred	Hundredth
12	Twelve	Twelfth (12 th)	1000	One thousand	Thousandth
13	Thirteen	Thirteenth (13 th)	100000	One hundred thousand	Hundred thousandth
20	Twenty	Twentieth (20 th)	1000000	One million	Millionth

Expressing numbers in English:

* If a number is between 21 and 99, and the second digit is not zero, we should write the number as two words separated by a **hyphen**.

Examples: 34 ≡ thirty-four; 62 ≡ sixty-two; 98 ≡ ninety-eight

* If a number is over 100 it is usually written in figures. However, if you want to write it in words you must put the word **“and”** before the last two figures.

Examples: 325 ≡ three hundred **and** twenty-five; 709 ≡ seven hundred **and** nine

* If a number is between 1000 and 1.000.000 is usually written **using commas**.

Examples: 1.306 ≡ one thousand, three hundred and six
 4.512 ≡ four thousand, five hundred and twelve
 6.305.215 ≡ six million, three hundred and five thousand, two hundred and fifteen

Remember that in English they use commas instead of points to express numbers greater than 999

Note :

Generally speaking, the word **“and”** is used after every hundred digit of a number. **Examples:**

512 ≡ five hundred **and** twelve
 2.340.000 ≡ two million, three hundred **and** forty thousand

They don't put “and” in American English!

* If a number is a four-figure number ending in 00 it could be said or sometimes written as a number of hundreds.

Examples: 1.600 \equiv “sixteen hundred”

2.700 \equiv “twenty-seven hundred”

Special cases

2 \equiv couple or pair

6 \equiv half a dozen

12 \equiv a dozen (used mostly in commerce)

1.000.000.000.000 \equiv billion

Note :

Long time ago, British speakers used " **billion** " to mean a million million like in Spanish. It means **1.000.000.000.000**

However, nowadays they usually use it to mean a **thousand million** (or milliard), like American speakers. It means **1.000.000.000**

Saying telephone numbers

Each number is usually said separately. If a telephone number contains a double number, we use the word “double”. Sometimes we group them and introduce pauses in between.

Example: 644 012 839 \equiv six four four “oh” one two eight three nine

Saying time of day

Let’s see some **example :**

7:00 a.m. *or* seven o’clock in the morning

3:30 p.m. *or* half past three in the afternoon

8:15 p.m. *or* a quarter past eight in the evening



Saying dates

The day usually comes before the month and the ordinal suffix is always vocalised and most of the times it is appended.

Example: 2nd May 1967

Curiously, when we write it down or when we say it we normally put “the” and “of” like:

Example: The second of May ...

Years

A year is usually said in two parts.

Examples: 1997 \equiv nineteen ninety-seven

2015 \equiv twenty fifteen

★ Except 2000 which is normally said *two thousand*

* If the year ends in *01* to *09* we usually say it like a number.

Example: 2004 \equiv two thousand and four

If the year ends in 00
we usually say *hundred*

Percentages

You can say “per cent” or “percent” after the number.

Example: 67% \equiv sixty seven per cent

Fractions

There are several ways to say a fraction in English.

Example: $1/8 \equiv$ one over eight

one – eighth

one *divided by* eight

You use ordinals for the
denominator and place a
hyphen in between.

* If it is necessary we use **plural** in the second part.

Examples: $3/6 \equiv$ three sixths

$2/3 \equiv$ two thirds

* There are some special cases:

$1/2 \equiv$ one half *or* a half

What is more, we can say it every time it appears. For instance:

$4\frac{1}{2} \equiv$ four and a half

$1/4 \equiv$ a quarter

Decimal numbers

➤ In decimal numbers, remember that:

- In Spain, we use a comma and in Britain, they use a point.
- After a comma, every digit is said separately.
- The figure 0 is usually called *nought* before a comma and *oh* after the comma. **Example:** 0,203 \equiv nought comma two oh three

Remember that in English they use points instead
of commas to express decimal numbers