

MATHEMATICS 2 ESO

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PREFACIO

“La esencia de las Matemáticas no consiste en complicar lo que es simple, sino en simplificar lo que es complicado”.

Stanley Gudder

La enseñanza de las Matemáticas en la Educación Secundaria, lo sabemos bien los que nos dedicamos a ello, puede llegar a ser una tarea difícil y más aún si se realiza en una lengua que no es la lengua materna del alumnado.

Este libro surge precisamente de mi interés por elaborar un material sencillo y atractivo para el estudiante y que al mismo tiempo sea fiel al currículo de esta etapa. El hecho es que facilita muchísimo la labor del profesorado pues se trata de un compendio de contenidos, actividades y problemas que puede ser utilizado perfectamente como libro de texto, lo que es una gran ventaja como he podido comprobar personalmente en mi labor como profesora.

El libro está estructurado en unidades didácticas. Cada una de ellas consta de:

- Índice detallado.
- “Key Vocabulary”, es decir, el **vocabulario técnico de Matemáticas en inglés** que necesita el alumnado específicamente para cada unidad.
- Esquema introductorio, que informa de lo que se va a estudiar en la unidad correspondiente.
- **Contenidos**, que es la parte esencial de la misma y que está plagada de ejemplos, casos particulares, diagramas, gráficos... todo ello elaborado cuidando el detalle y el color.
- Una **tabla** donde se recoge el vocabulario nuevo aprendido y que el alumno debe completar con la fonética de cada palabra y su significado.
- Y por último se incluyen una **amplia colección de actividades, ejercicios y problemas de cada unidad didáctica**.

Se trata de un material adaptable a diversas metodologías pues está elaborado teniendo en cuenta los principios de la metodología AICLE (CLIL) que ofrece la posibilidad de aprender los contenidos curriculares de la asignatura de Matemáticas a la vez que permite practicar la lengua inglesa aprendida en etapas previas incrementando su bagaje de vocabulario técnico específico de Matemáticas en Inglés.

El hecho de que esté íntegramente elaborado en inglés constituye una ventaja para la inmersión total del alumnado en la lengua inglesa.

El alumno asimila palabras, frases y vocabulario cotidiano de la lengua inglesa además de estructuras y vocabulario específico de la propia asignatura de Matemáticas.

Es más, como es el propio alumno el que completa la tabla fonética de cada unidad, va interiorizándola sin apenas darse cuenta; tabla que puede ampliar con los términos que cada uno decida individualmente. Se trata en definitiva de un material que el alumno mismo ayuda a elaborar según sus necesidades particulares y que al final del año escolar le habrá servido para confeccionar su propia lista de vocabulario específico.

Otra ventaja de estos materiales es que por la **sencillez y concreción** con que están definidos los conceptos en el libro, se facilita el aprendizaje y se produce un impacto en la conceptualización, es decir, el alumno llega a ser capaz de pensar directamente en lo que se dice aunque esté expresado en otra lengua centrándose en los contenidos curriculares de la materia. Este aspecto ayuda a ampliar su mapa conceptual del pensamiento y a desarrollar en mayor medida sus competencias.

Y si alguna cosa más hubiera que destacar, personalmente destacaría la **motivación** que se logra en el alumnado al trabajar con este material. Por su estructura práctica, el colorido elegido al detalle para hacerlo atractivo, las listas de recursos web que se facilitan, porque las actividades y problemas planteados se ajustan a los contenidos y al nivel con propiedad, o por todo ello unido, se produce un efecto participativo y motivador que de otra forma es difícil alcanzar en la actualidad.

Por último decir que este libro está escrito con la ilusión de compartir y transmitir dos de mis grandes pasiones, la lengua inglesa y las Matemáticas, de la manera más sencilla posible.

ROSARIO CARRASCO TORRES

MATHEMATICS

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	CARDINAL	ORDINAL		CARDINAL	ORDINAL
1	One	First (1 st)	21	Twenty-one	Twenty-first (21 st)
2	Two	Second (2 nd)	22	Twenty-two	Twenty-second (22 nd)
3	Three	Third (3 rd)	23	Twenty-three	Twenty-third (23 rd)
4	Four	Fourth (4 th)	24	Twenty-four	Twenty-fourth (24 th)
5	Five	Fifth (5 th)	25	Twenty-five	Twenty-fifth (25 th)
6	Six	Sixth (6 th)	26	Twenty-six	Twenty-sixth (26 th)
7	Seven	Seventh (7 th)	27	Twenty-seven	Twenty-seventh (27 th)
8	Eight	Eighth (8 th)	28	Twenty-eight	Twenty-eighth (28 th)
9	Nine	Ninth (9 th)	29	Twenty-nine	Twenty-ninth (29 th)
10	Ten	Tenth (10 th)	30	Thirty	Thirtieth (30 th)
11	Eleven	Eleventh (11 th)	40	Forty	Fortieth (40 th)
12	Twelve	Twelfth (12 th)	50	Fifty	Fiftieth (50 th)
13	Thirteen	Thirteenth (13 th)	60	Sixty	Sixtieth (60 th)
14	Fourteen	Fourteenth (14 th)	70	Seventy	Seventieth (70 th)
15	Fifteen	Fifteenth (15 th)	80	Eighty	Eightieth (80 th)
16	Sixteen	Sixteenth (16 th)	90	Ninety	Ninetieth (90 th)
17	Seventeen	Seventeenth (17 th)	100	One hundred	Hundredth
18	Eighteen	Eighteenth (18 th)	1000	One thousand	Thousandth
19	Nineteen	Nineteenth (19 th)	100000	One hundred thousand	Hundred thousandth
20	Twenty	Twentieth (20 th)	1000000	One million	Millionth

REMEMBER THAT:

- In decimal numbers we use a **comma** and in Britain they use a point.
- The figure 0 is usually called **nought** before a comma and **oh** after the comma. After the comma every digit is said **separately**.
Example: “0,30014 ≡ nought comma three oh oh one four”
- In team games, zero scores are usually called **nil**.
Example: “Spain two France nil”
- For the number **0** or for the temperatures we say **zero**.
Example: “-5°C ≡ five degrees below zero”
- “**and**” is used after any hundred digit of a number.
Examples: “214 ≡ two hundred and fourteen”; “502 ≡ five hundred and two”.
- Numbers are normally written in singular but you can use plural with hundred, thousand, million... if they are followed by “of”.
Examples: “Thousands of years ago...” ; “There are hundreds of insects...”
- **Fractions:** there are several ways to say a fraction.
Examples: $2/3 \rightarrow$

{	two over three
	two thirds
	two divided by 3

 $9/7 \rightarrow$

{	nine over seven
	nine sevenths
	nine divided by seven

The particular case of **2** as denominator: $\frac{1}{2} \rightarrow$ one half / a half

UNIT 1: “INTEGERS”

1. Integer numbers
 - 1.1. Order and representation
 - 1.2. Absolute value
 - 1.3. Opposite number
2. Operations
 - 2.1. Addition and subtraction
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5. Divisibility
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 - 5.4. L.C.M. (Lowest Common Multiple)

KEY VOCABULARY:

Positive number
 Plus
 Negative number
 Minus
 Integer (or whole number)
 Order
 Greater than
 Smaller than
 Absolute value
 Opposite
 Addition
 To add
 Subtraction
 To subtract
 To multiply
 To divide
 Factor of
 Prime number
 Power
 Base
 Exponent
 Even
 Odd
 Parenthesis
 Brackets
 Square root
 Radical
 Radicand
 Index / Indices
 Hierarchy
 Divisible by
 Divisor of
 Remainder
 Composite number
 G.C.D. or G.C.F.
 L.C.M.

In this unit you will learn how to:

- *Order Integer numbers*
- *Operate with integer numbers*
- *Calculate the lowest common multiple and the highest common factor of some numbers*
- *Apply the properties of powers*
- *Express roots and powers*

1. Integer numbers

There are times when we can't express or solve some situations with the natural numbers (\mathbb{N}) and we need to enlarge that set to the integer numbers set (\mathbb{Z}).

The set of the integers is formed by:

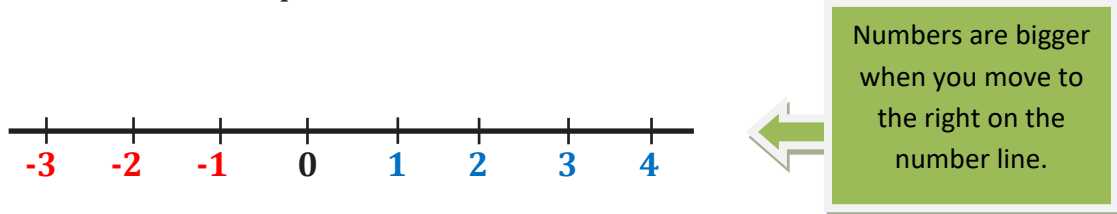
$\left\{ \begin{array}{l} \text{positive integers: } + 1, + 2, + 3 \dots \\ \text{zero: } 0 \\ \text{negative integers: } - 1, - 2, - 3 \dots \end{array} \right.$

REMEMBER
THAT:
 $+ 2 = 2$

plus
minus

1.1. Order and representation

The integer numbers can be represented in the number line like that:



We can use symbols $<$ and $>$ to express which is the order relation between two numbers. For instance we can write: $-2 < -1$ $7 > -5$ $0 < 9$ $-6 > -8$

"-2 is smaller than -1"

$-2 < -1$

$7 > -5$

"7 is greater than -5"

1.2. Absolute value

The absolute value of a number x is the number we obtain when we eliminate the negative sign of the number. It can be written $|x|$.

Examples: $|2| = 2$ $|-6| = 6$

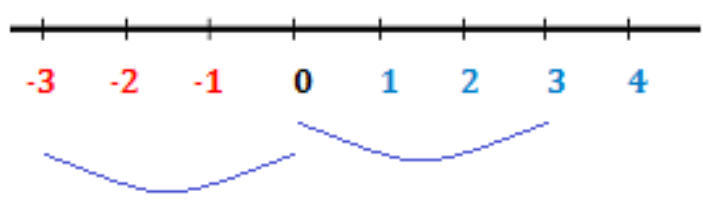
We can say that: the absolute value of -8 is 8.

1.3. Opposite number

The opposite of a number x is $-x$. It means that we must change the sign of the number. They both have the same absolute value but their signs are different. We can also represent $Op(x) = -x$.

Examples: $Op(11) = -11$ $Op(-3) = 3$

You can notice that two opposite numbers are at equal distance to the number zero in the number line.



2. Operations

2.1. Addition and subtraction

Addition:

- a) If the numbers have the same sign then add the absolute values of the numbers and write the sign they have. **Example:** $(-3) + (-5) = -8$
- b) If the numbers have different sign then subtract the absolute values of the numbers and write the sign of the one that has the highest absolute value. **Example:** $(-3) + (+5) = +2$

Subtraction:

To subtract integer numbers you must add the minuend to the opposite of the subtrahend. **Example:** $(-8) - (+6) = -14$

Remember the rule of the signs:

$$\begin{aligned} (+) \cdot (+) &= + & (+) \cdot (-) &= - \\ (-) \cdot (-) &= + & (-) \cdot (+) &= - \end{aligned}$$

2.2. Multiplication and division

To multiply and divide we must write the sign of the result following the rule of the signs and then we must do the operation as always.

Examples: $(-7) \cdot (+3) = -21$
 $(-8) : (-4) = +2$

← THE SAME RULE FOR DIVISIONS

2.3. Powers

A power is the shortest way to express a product of a number by itself repeatedly.

$$a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{(n \text{ times})}$$

a is called the **base**
n is called the **exponent**

Example: 2^4 This expression can be read as: "Two raised to the fourth power"

5^2 can be read five squared
 7^3 can be read seven cubed

- "Two to the fourth power"
- "Two to the fourth"
- "Two to the power of four"

The most used

2.3.1. Expressions

SIGNS OF POWERS

* **EVEN** exponent \Rightarrow **positive power** **Example:** $(+2)^4 = 16$
Example: $(-2)^4 = 16$

* **ODD** exponent:
 a) **IF THE BASE IS POSITIVE** \Rightarrow **positive power** **Example:** $(+2)^3 = 8$
 b) **IF THE BASE IS NEGATIVE** \Rightarrow **negative power** **Example:** $(-2)^3 = -8$

Take a look:

$$(+2) \cdot (+2) \cdot (+2) = 2 \cdot 2 \cdot 2 = 2^3$$

$$(-7) \cdot (-7) \cdot (-7) \cdot (-7) = (-7)^4 = 7^4$$

Surprisingly: $-7^4 \neq 7^4$

POWERS OF TEN

If we work with big numbers or very small numbers it is preferable to use powers of ten as in the examples:

$$876\,000\,000\,000\,000 = 876 \cdot 10^{12}$$

$$0,000\,005 = 5 \cdot 10^{-6}$$

2.3.2. Properties of powers

- The product of powers with the same base is another power with the same base and the exponent is the addition of the exponents.

$$a^n \cdot a^m = a^{n+m}$$

$$2^3 \cdot 2^4 = 2^7$$

- The quotient of powers with the same base is another power with the same base and the exponent is the subtraction of the exponents.

$$a^n : a^m = a^{n-m}$$

$$2^5 : 2^2 = 2^3$$

- The power of a power is another power with the same base and the exponent is the product of the exponents.

$$(a^n)^m = a^{n \cdot m}$$

$$(2^5)^3 = 2^{15}$$

- The power of a product (quotient) is the product (quotient) of the powers of the factors (numerator and denominator).

$$(a \cdot b)^n = a^n \cdot b^n$$

$$(7 \cdot 5)^2 = 7^2 \cdot 5^2$$

$$(a : b)^n = a^n : b^n$$

$$(15 : 5)^2 = 15^2 : 5^2$$

REMEMBER THAT:

$$a^0 = 1$$

$$a^1 = a$$

3. Square roots

The **square root** of a number **a** is another number **b** that multiplied by itself gives the number **a**. It means that: $\sqrt{a} = b \leftrightarrow b^2 = a$

The symbol $\sqrt{\quad}$ is called **radical symbol** and the number **a** is called **radicand**. We also can use **radical** to refer to \sqrt{a} .

Example: $\sqrt{9} = 3$ since $3^2 = 9$

$$\sqrt{-n} \nexists, n > 0$$

4. Order or hierarchy of the operations (Bidmas)

The hierarchy of the operations between integer numbers is usually remembered thanks to a mnemonic rule called: **BIDMAS**

Brackets - **Indices** - **Divisions** - **Multiplications** - **Additions** - **Subtractions**

The order of operations is one of the golden rules.

Take a look:

$$8 - 4 \cdot 6 = -16$$

Example: $[-11 - 5 \cdot (-3)^2] : \sqrt{64} = [-11 - 45] : 8 = (-56) : 8 = -7$

5. Divisibility

➤ A division is **exact** when the remainder is 0. Example:

$$\begin{array}{r|l} 10 & 5 \\ 0 & 2 \end{array}$$

In that case we can say that:

- 10 is a **multiple of 5**
- 10 is **divisible by 5**
- 5 is a **divisor of 10**
- 5 is a **factor of 10**

➤ A division is **not exact** when the remainder is not 0.

Generally speaking we can say:

a is multiple of b
b is a divisor of a

The set of multiples of a number **a** can be expressed by \dot{a} .

Example: $\dot{5} = \{ \dots, -15, -10, -5, 0, 5, 10, 15, 25, \dots \}$

➤ If a number can be divided only by 1 and by itself then it is a **prime number**.

Examples: 7, 2, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, ...

➤ Integer numbers that have more than two divisors are called **composite numbers**. Examples: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, ...

5.1. Rules of divisibility

RULES OF DIVISIBILITY	The number is divisible by:
The last digit is 0 or even	2
The sum of the digits is divisible by 3	3
The last digit is 0 or 5	5
The sum of the digits is divisible by 9	9
The last digit is 0	10
The (sum of the odd positioned digits) - (sum of the even positioned digits) is divisible by 0 or 11. Example: 34871903 $3 + 8 + 1 + 0 = 12$ $4 + 7 + 9 + 3 = 23$ $23 - 12 = 11$ Then the number is divisible by 11	11
TAKE A LOOK: If a number is divisible by two different prime numbers, then it is divisible by the product of those two numbers. Since 12, is divisible by both 2 and 3, it is also divisible by 6.	

5.2. Prime factorization

The process of finding the prime numbers that divide exactly an integer number is called **prime factorization** or **integer factorization**.

$$\begin{array}{r|l}
 24 & 2 \\
 12 & 2 \\
 6 & 2 \\
 3 & 3 \\
 1 &
 \end{array}$$

$$24 = 2^3 \cdot 3$$

$$\begin{array}{r|l}
 42 & 2 \\
 21 & 3 \\
 7 & 7 \\
 1 &
 \end{array}$$

$$-42 = (-1) \cdot 2 \cdot 3 \cdot 7$$

When the number is negative we add - 1 to the prime factorization.

5.3. G.C.D. (Greatest Common Divisor or Factor G.C.F.)

The Greatest Common divisor of several numbers is the largest of their common divisors.

To calculate the G.C.D.:

First step: calculate the prime factorization of the numbers

Second step: choose the **common factors** raised to the **smallest exponent**

TAKE A LOOK:
G.C.D. (15, - 27) = G .C.D. (15, 27) = 3

Example:

Calculate the G.C.D. (45, 24, 12)

- $45 = 3^2 \cdot 5$
- $24 = 2^3 \cdot 3$
- $12 = 2^2 \cdot 3$
- G.C.D. (45, 24, 12) = 3

Special case: G.C.D. (a, b) = 1 if a and b don't have any common divisor.

Example: G.C.D. (25, 12) = 1