

MATHEMATICS

2º ESO

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INDEX

LESSON 1: DIVISIBILITY AND INTEGERS	5
1. FACTORS AND MULTIPLES	5
2. DIVISIBILITY RULES	5
3. PRIME AND COMPOSITE NUMBERS	6
4. PRIME FACTOR DECOMPOSITION OF A NUMBER	6
5. LOWEST COMMON MULTIPLE (L.C.M.)	7
6. HIGHEST COMMON FACTOR (H.C.F.)	7
7. ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION WITH INTEGERS	7
8. POWERS OF INTEGERS	8
9. SQUARE ROOT OF INTEGERS	9
Worksheet	10
LESSON 2: DECIMAL SYSTEM AND SEXAGESIMAL SYSTEM	12
1. DECIMAL SYSTEM	12
2. TYPES OF DECIMAL NUMBERS	12
3. DECIMAL NUMBERS ON THE NUMBER LINE	13
4. ROUNDING DECIMAL NUMBERS	13
5. SEXAGESIMAL SYSTEM	13
6. OPERATIONS IN THE SEXAGESIMAL SYSTEM	14
Worksheet	16
LESSON 3: FRACTIONS	18
1. THE MEANING OF FRACTIONS	18
2. EQUIVALENT FRACTIONS	18
3. SIMPLIFYING FRACTIONS	19
4. COMMON DENOMINATOR	19
5. ADDITION AND SUBTRACTION OF FRACTIONS	20
6. MULTIPLICATION AND DIVISION OF FRACTIONS	20
7. POWERS OF FRACTIONS	20
8. FRACTIONS AND DECIMALS	21
Worksheet	23
LESSON 4: PROPORTIONALITY AND PERCENTAGES	25
1. RATIOS AND PROPORTIONS	25
2. DIRECT PROPORTION	25
3. INVERSE PROPORTION	26
4. MORE THAN ONE PROPORTIONAL RELATIONSHIP	26
5. PERCENTAGES	27
Worksheet	29
LESSON 5: ALGEBRA	33
1. ALGEBRA (WHAT IS IT FOR?)	33
2. ALGEBRAIC EXPRESSIONS	33
3. EXPANDING AND FACTORING ALGEBRAIC EXPRESSIONS	37
4. SPECIAL PRODUCTS	37
Worksheet	38
LESSON 6: EQUATIONS	41
1. EQUATIONS	41
2. SOLVING LINEAR EQUATIONS	41
3. QUADRATIC EQUATIONS	42
Worksheet	44

LESSON 7: SYSTEMS OF LINEAR EQUATIONS	46
1. LINEAR EQUATIONS WITH TWO UNKNOWNNS	46
2. GRAPHING LINEAR EQUATIONS	46
3. SYSTEMS OF LINEAR EQUATIONS	48
4. SOLVING SYSTEMS OF LINEAR EQUATIONS BY GRAPHING	48
5. ALGEBRAIC METHODS TO SOLVE SYSTEMS OF LINEAR EQUATIONS	50
Worksheet	53
LESSON 8: FUNCTIONS	55
1. FUNCTIONS	55
2. INCREASING AND DECREASING FUNCTIONS	56
3. CONTINUOUS AND DISCONTINUOUS FUNCTIONS	57
4. GRAPHING A FUNCTION GIVEN A FORMULA	57
5. LINEAR GRAPHS	58
Worksheet	61
LESSON 9: STATISTICS	64
1. STATISTICS	64
2. FREQUENCY	64
3. PARAMETER STATISTICS	65
4. OTHER GRAPHS	66
Worksheet	67
LESSON 10: PYTHAGORAS THEOREM. SIMILARITY	68
1. TRIANGLES	68
2. PYTHAGORAS THEOREM	70
3. SIMILAR SHAPES	70
4. MAPS	71
5. THALES THEOREM (INTERCEPT THEOREM)	71
6. TRIANGLES IN THE THALES POSITION	71
7. SIMILAR TRIANGLES. SIMILARITY CRITERIA	71
Worksheet	73
LESSON 11: 3D-SHAPES	75
1. POLYHEDRONS	75
2. PRISMS	75
3. PYRAMIDS	76
4. CYLINDER	77
5. CONE	77
6. SPHERE	78
Worksheet	79

LESSON 1: DIVISIBILITY AND INTEGERS

<u>Keywords</u>			
Multiple	Prime	Composite	LOWEST COMMON MULTIPLE
<i>Factor</i>		Highest Common factor	BODMAS
Powers	BASE	EXPONENT	Square Root

1. FACTORS AND MULTIPLES

Factors of a natural number are numbers that divide exactly into it (with zero as remainder).

Example: The factors of 12 are 1, 2, 3, 4, 6 and 12. (If you divide 12 by them you get remainder zero)

Multiples of a natural number are numbers that you get multiplying it.

Example: 3, 6, 9, 12, ... are multiples of 3 because

$$\begin{aligned} 3 &= 3 \cdot 1 \\ 6 &= 3 \cdot 2 \\ 9 &= 3 \cdot 3 \\ 12 &= 3 \cdot 4 \end{aligned}$$

Question: Is 46 a multiple of 5?

Observe:

1. If 3 is a factor of 12, 12 is a multiple of 3, and we say that 12 is divisible by 3.

$$\begin{array}{r|l} a & b \\ 0 & c \end{array}$$

- ◆ a is divisible by b
- ◆ b is a factor of a
- ◆ c is a factor of a
- ◆ a is multiple of b

2. a is multiple of a .
3. a is a factor of a .
4. 1 is a factor of all numbers.
5. A number has infinite multiples.
6. If you add two multiples of n , the result is a multiple of n .

Example:

$$\left. \begin{array}{l} 6 \text{ is a multiple of } 3 \\ 9 \text{ is a multiple of } 3 \end{array} \right\} \implies 6 + 9 = 15 \text{ is a multiple of } 3$$

2. DIVISIBILITY RULES

2.1. Divisibility by 2

A number is divisible by 2 if it ends in an even digit (0, 2, 4, 6 or 8).

Examples: 2, 4, 6, 8754, 986432... are divisible by 2.

2.2. Divisibility by 3

How to know if a number is divisible by 3:

1. Add up all the digits in the number.
2. If the result is multiple of 3, the number is divisible by 3.

Example: 5415 is divisible by 3 ($5 + 4 + 1 + 5 = 15$ multiple of 3)

2.3. Divisibility by 5

Numbers ending in a 5 or a 0 are divisible by 5.

Example: 876435 is divisible by 5 (it ends in a 5).

2.4. Divisibility by 10

Numbers ending in a 0 are divisible by 10.

Example: 8754330 is divisible by 10.

3. PRIME AND COMPOSITE NUMBERS

- A **prime number** is a natural number that has exactly two factors: 1 and itself.
Note: 1 is not considered to be a prime number.
- A **composite number** is a natural number that has more than two factors. It is a product of prime numbers.

Exercise: The "Sieve of Erathostenes" is a procedure to find the first prime numbers. Use it to find out the prime numbers less than 50.

4. PRIME FACTOR DESCOMPOSITION OF A NUMBER

The **factoring of a number** means writing it as a product of prime factors (using powers).

How to factor a number:

1. Try to divide the number by the first prime number, which is two. If you get remainder zero try to divide by two again, if not, divide by the following prime number, 3.
2. Repeat step 1 with the following prime numbers until the final quotient is 1.

Example: Factor 72.

72	2	So, we have the factoring: $72 = 2^3 \cdot 3^2$
36	2	
18	2	
9	3	
3	3	
1	1	

Note: Factors of a number are obtained by multiplying its prime factors.

5. LOWEST COMMON MULTIPLE (LCM)

It is the smallest common multiple of two or more numbers (you can divide the LCM of two numbers by both of them exactly).

How to calculate LCM:

- Factor the numbers.
- Choose **non-common and common** factors with the **highest exponents**.
- The LCM is the product of those powers.

Example: Find the LCM of 36 and 40.

$$\begin{array}{r|l} 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

$$36 = 2^2 \cdot 3^2$$

$$\begin{array}{r|l} 40 & 2 \\ 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

$$40 = 2^3 \cdot 5$$

$$\text{LCM}(36, 40) = 2^3 \cdot 3^2 \cdot 5 = 360$$

6. HIGHEST COMMON FACTOR (HCF)

It is the largest number that is a factor of all of them at the same time.

How to calculate the HCF:

- Factor the numbers.
- Choose **common** factors with the **lowest exponents**.
- The HCF is the product of those powers.

Example: Find the HCF of 36 and 40.

$$\begin{array}{r|l} 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

$$36 = 2^2 \cdot 3^2$$

$$\begin{array}{r|l} 40 & 2 \\ 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

$$40 = 2^3 \cdot 5$$

$$\text{HCF}(36, 40) = 2^2 = 4$$

7. ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION WITH INTEGERS

7.1. Addition

- Integers with the **same sign**: you keep the sign and add the numbers.

Examples:

a) $(+3) + (+5) = (+8)$

b) $(-4) + (-7) = (-11)$

- Integers with **different sign**: you choose the sign of the highest number and subtract them.

Examples:

a) $(+9) + (-4) = (+5)$

b) $(+4) + (-15) = (-11)$

7.2. **Subtraction**

To subtract integers change the sign of the number that is to be subtracted, then you do it like additions.

Examples:

a) $(-3) - (+7) = -3 - 7 = -10$

b) $5 - (-8) = 5 + 8 = 13$

7.3. **Multiplication**

- First you have to multiply signs.

$$(+)\cdot(+)=(+)$$

$$(+)\cdot(-)=(-)$$

$$(-)\cdot(+)=(-)$$

$$(-)\cdot(-)=(+)$$

- Then, you multiply numbers.

Examples:

a) $(-3)\cdot(+6) = (-18)$

b) $(-5)\cdot(-9) = +45$

7.4. **Division**

- First you have to divide signs.

$$(+):(+)=(+)$$

$$(+):(-)=(-)$$

$$(-):(+)=(-)$$

$$(-):(-)=(+)$$

- Then, you divide numbers.

Examples:

a) $(-14):(+7) = -2$

b) $(-55):(-5) = +11$

8. **POWERS OF INTEGERS**

$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ Two to the power of 5 or two to the fifth
 Base $\longrightarrow 2^5 \longleftarrow$ Exponent

Properties:

- $a^0 = 1$
- $a^1 = a$

- The power of a product is the product of the powers.

Example: $(2 \cdot 3)^4 = 2^4 \cdot 3^4$

$$(a \cdot b)^n = a^n \cdot b^n$$

- The power of a quotient is the quotient of the powers.

Example: $(2 : 3)^4 = 2^4 : 3^4$

$$(a : b)^n = a^n : b^n$$

5. When multiplying powers *of the same base*, you keep the same base and add the exponents.

Example: $5^3 \cdot 5^4 = 5^7$

$$a^n \cdot a^m = a^{n+m}$$

6. When dividing powers *of the same base*, you keep the same base and subtract the exponents.

Example: $7^8 : 7^5 = 7^3$

$$a^n : a^m = a^{n-m}$$

7. When powering a power, you keep the base and multiply the exponents.

Example: $(4^3)^2 = 4^6$

$$(a^n)^m = a^{n \cdot m}$$

8. If the base is a negative number there are two options:
 - If the exponent is an even number, the result is a positive number.
 - If the exponent is an odd number, the result is a negative number.

Examples:

a) $(+7)^3 = 7 \cdot 7 \cdot 7 = 343$

b) $(-5)^2 = +25$

c) $(-3)^3 = -27$

9. SQUARE ROOT OF INTEGERS

$$\sqrt{a} = b \implies b^2 = a \quad (a \text{ radicand, } b \text{ root})$$

The opposite of squaring a number is calculating its square root.

Example: $\sqrt{9} = 3$ The square root of nine equals 3

- ✓ The square root of a negative number doesn't exist.
- ✓ The square root of a positive number has two solutions, one positive and one negative.

Example:

$$\sqrt{16} = \pm 4 \quad \text{because } (+4)^2 = 16 \quad \text{and } (-4)^2 = 16$$

WORKSHEET

Exercise 1. Each of these calculations has an answer which is one of the numbers of the list: 12, -12, -3, -4, 3, 8.

Select the correct number for each answer:

a) $3 \cdot (-4) = \dots\dots$

d) $4 + (-7) = \dots\dots$

b) $-16 \div (-2) = \dots\dots$

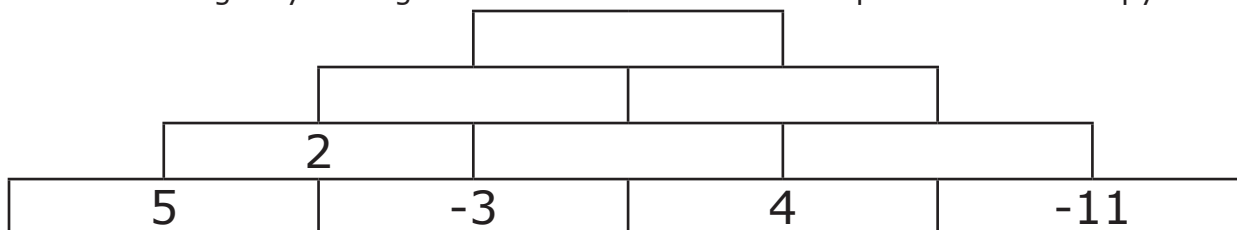
e) $-9 \div (-3) = \dots\dots$

c) $8 - (-4) = \dots\dots$

f) $-7 - (-3) = \dots\dots$

Exercise 2. The temperature at midday is -2°C . By midnight it has dropped 10 degrees. What is the temperature at midnight?

Exercise 3. Here are some numbers in a number pyramid. You calculate the number in each rectangle by adding the two numbers below. Complete the number pyramid.



Exercise 4. Here are some signs:



Insert the correct sign to make each calculation correct:

a) $-12 \dots (-3) = 4$

c) $-7 \dots (-3) = -4$

b) $3 \dots (-3) = -9$

d) $12 \dots (-5) = 7$

Exercise 5. Here is a list of numbers: -9, -5, -1, 0, 3, 2, 5

Choose the number from the list to make each sentence correct:

a) $3 - \dots = 4$

d) $2 - \dots = 7$

b) $5 \cdot \dots = -25$

e) $-9 \div \dots = -3$

c) $-9 + \dots = -4$

Exercise 6. A farmer can make groups of 5, 6 or 9 hens without leaving any of his hens out. There are less than 100 hens in this farm. How many hens are there in the farm?



Exercise 7. A cook in Iowa made a huge pizza on June 19th 2005 and he entered the Guinness book of records. It was a rectangular pizza which was 44 m. long and 33 m. wide.

We are going to divide this pizza into square portions which are as big as possible without throwing away any pizza.

- How long should each side of the pizza be?
- How many portions are we going to get?



Exercise 8. Two athletes A and B started running from the starting line of a stadium at the same time. Both of them ran at a constant speed and it took them 84 seconds and 96 seconds to finish a lap.

- How often did they cross the starting line together?
- How many laps had each of them completed when they crossed the starting line together for the first time after the race had started?

Exercise 9. We want to cut two wires which are 30 m and 66 m long into pieces of the same length that are as big as possible and without wasting any wire. How long is each piece?



LESSON 2: DECIMAL SYSTEM AND SEXAGESIMAL SYSTEM

Exact decimal Irrational number Sexagesimal system	<p style="margin: 0;"><u>Keywords</u></p> Repeating decimal ROUNDING digit	Point PERIOD NUMBER LINE DEGREES
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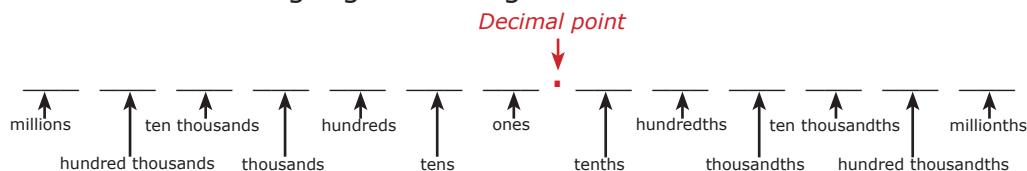
1. DECIMAL SYSTEM

A **Decimal number** has two parts separated by a decimal point.

Example: 13 is the whole part; 147 is the decimal part
 We read 13.147 as "thirteen, point, one, four, seven".

Nowadays, we often use decimal system. For example, we read 5.24 € as "five point twenty four euros" or "five euros and twenty four cents" or we read 5.36 m as "five point thirty six metres".

Decimal system is a base 10 system: on this system every unit is divided into 10 equal parts to get the subunit. So, each digit has a value which is 10 times larger than the following digit on its right.



2. TYPES OF DECIMAL NUMBERS

There are three types of decimal numbers:

- **Exact Decimals (or Terminating Decimals).** -There is a limited quantity of digits in the decimal part.

Example: 13.121

- **Repeating Decimals (or Recurring Decimals).** -There is an unlimited quantity of digits in the decimal part and there is a group of digits which are repeated forever.

Examples: If you divide 1 by 3 you get 0.3333333...
 If you divide 5 by 22 you get 0.2272727...

The group of repeated decimal digits is called **period**. We usually write $\overline{\quad}$ over the period.

Examples: $0.3333333... = 0.\overline{3}$ (the period is 3)
 $0.2272727... = 0.2\overline{27}$ (the period is 27)

- **Irrational Numbers.** There is an unlimited quantity of digits in the decimal part but there is not any period.

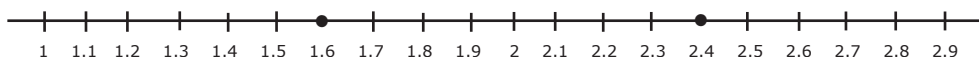
Examples: Calculating $\sqrt{2}$ we get 1.414213562... (it is an irrational number)

$\pi = 3.141592653589793238462643...$ (it is an irrational number)

3. DECIMAL NUMBERS ON THE NUMBER LINE

Every decimal number has a place on the number line between two integer numbers.

Example: We can represent the numbers 1.6 or 2.4 dividing the units into ten equal parts:



- Property: Between two decimal numbers there are infinite decimal numbers.

Examples:

Between 1.6 and 1.7 we can find, for example, 1.63 and $1.6 < 1.63 < 1.7$

Between 1.63 and 1.64 we can find, for example, 1.637 and $1.63 < 1.637 < 1.64$

4. ROUNDING DECIMAL NUMBERS

To round a number to any decimal place value we look at the digit to the right of the place we wish to round to and when the digit 5, 6, 7, 8, or 9, appears in that place, you must add one unit to the last digit; when the digit 0, 1, 2, 3, or 4 appears in that place, you must cut the number.

Examples:

1.38 rounded to the nearest tenth is 1.4

2.36321 rounded to the nearest hundredth is 2.36

5. SEXAGESIMAL SYSTEM

We frequently use sexagesimal system when we talk about angles and time. In sexagesimal system every unit is divided into 60 equal parts to get the sub-unit.

- Angles:

The unit is the degree ($^{\circ}$). The subunits are minutes ($'$) and seconds ($''$).

One minute: $1' = \frac{1}{60}$ of a degree. That is, $60' = 1^{\circ}$

One second: $1'' = \frac{1}{60}$ of a minute. That is, $60'' = 1'$

Example: An angle "A" is expressed as $A = 43^{\circ}12'5''$

-Time:

The unit is the hour (h). The subunits are minutes (min) and seconds (s).

One minute: $1 \text{ min} = \frac{1}{60}$ of an hour. That is, $60 \text{ min} = 1 \text{ h}$

One second: $1 \text{ s} = \frac{1}{60}$ of a minute. That is, $60 \text{ s} = 1 \text{ min}$

Example: A period of time is expressed as 3 h 5 min 3 s.

6. OPERATIONS IN THE SEXAGESIMAL SYSTEM

6.1. Addition

We need to add degrees (or hours), minutes and seconds separately and then convert the seconds into minutes and the minutes into degrees (or hours) if we get more than 60 subunits.

Example: Add $45^\circ 53' 40'' + 12^\circ 33' 35''$

Adding separately we get $45^\circ 53' 40'' + 12^\circ 33' 35'' = 57^\circ 86' 75''$

but $75'' = 1' 15''$ so we add 1' and get $87' = 1^\circ 27'$ we add 1° and finally the solution is $57^\circ 86' 75'' = 58^\circ 27' 15''$

6.2. Subtraction

We need to subtract degrees (or hours), minutes and seconds separately, if we do not have enough seconds or minutes we convert one degree (or hour) into minutes or a minute into seconds.

Example: Subtract 57 h 13 min 21 s and 12 h 43 min 5s

We write 57 h 13 min 21 s as 56 h 73 min 21s and then we can subtract.

So, the solution is 45 h 30 min 16 s

6.3. Multiplication by a natural number

We multiply degrees minutes and seconds separately and then convert the seconds into minutes and the minutes into hours (or degrees) when we get more than 60 subunits.

Example: $(3 \text{ h } 22 \text{ min } 25 \text{ s}) \cdot 4$

$$\begin{array}{r}
 3 \text{ h} \quad 22 \text{ min} \quad 25 \text{ s} \\
 \cdot 4 \quad \cdot 4 \quad \cdot 4 \\
 \hline
 12 \text{ h} \quad 88 \text{ min} \quad 100 \text{ s} \\
 13 \text{ h} \quad 89 \text{ min} \quad \boxed{1 \text{ min}} 40 \text{ s} \\
 \quad \quad \quad \boxed{1 \text{ h}} 29 \text{ min}
 \end{array}$$

So, the solution is 13 h 29 min 40 s

6.4. Division by a natural number

We divide the hours (or degrees), and the remainder is converted into minutes that must be added to the previous quantity of minutes that we had, divide the minutes and we repeat the same that we have done before. The remainder is in seconds.

Example:

$$\begin{array}{r}
 46^{\circ} \quad 15' \quad 23'' \\
 2^{\circ} \cdot 60 = \underline{120'} \\
 135' \\
 3' \cdot 60 = \underline{180''} \\
 203'' \\
 3'' \longleftarrow \text{Remainder}
 \end{array}
 \qquad
 \begin{array}{r}
 4 \\
 \hline
 11^{\circ} 33' 50'' \longleftarrow \text{Quotient}
 \end{array}$$