educàlia
editorial

Rosario Carrasco Torres

MATHEMATICS 1 ESO

MATHEMATICS 1 ESO

Rosario Carrasco Torres

educàlia
editorial

Edición revisada: 2016

Autora: Rosario Carrasco Torres

Maquetación: Patricia Penavella Soto, Rosario Carrasco Torres

Imprime: Ulzama

ISBN: 978-84-941593-8-1

Depósito Legal: V-2109-2013

Printed in Spain/Impreso en España.

Educàlia Editorial S.L.
Avda. Jacarandas nº 2 - loft - 327 - 46100 Burjassot - València
Tel: 960 624 309 - 610 900 111 - 963 768 542
E-Mail: educaliaeditorial@e-ducalia.com
www.e-ducalia.com

PREFACIO

"La esencia de las Matemáticas no consiste en complicar lo que es simple, sino en simplificar lo que es complicado".

Stanley Gudder

La enseñanza de las Matemáticas en la Educación Secundaria, lo sabemos bien los que nos dedicamos a ello, puede llegar a ser una tarea difícil y más aún si se realiza en una lengua que no es la lengua materna del alumnado.

Este libro surge precisamente de mi interés por elaborar un material sencillo y atractivo para el estudiante y que al mismo tiempo sea fiel al currículo de esta etapa. El hecho es que facilita muchísimo la labor del profesorado pues se trata de un compendio de contenidos, actividades y problemas que puede ser utilizado perfectamente como libro de texto, lo que es una gran ventaja como he podido comprobar personalmente en mi labor como profesora.

El libro está estructurado en unidades didácticas. Cada una de ellas consta de:

- Índice detallado.
- "Key Vocabulary", es decir, el **vocabulario técnico de Matemáticas en inglés** que necesita el alumnado específicamente para cada unidad.
- Esquema introductorio, que informa de lo que se va a estudiar en la unidad correspondiente.
- **Contenidos**, que es la parte esencial de la misma y que está plagada de ejemplos, casos particulares, diagramas, gráficos… todo ello elaborado cuidando el detalle y el color.
- Una **lista de páginas web**, en inglés, donde los alumnos pueden practicar, experimentar e incluso aprender interactuando, los contenidos de la unidad.
- Una **tabla** donde se recoge el vocabulario nuevo aprendido y que el alumno debe completar con la fonética de cada palabra y su significado.
- Y por último se incluyen una **amplia colección de actividades, ejercicios y problemas de cada unidad didáctica.**

Se trata de un material adaptable a diversas metodologías pues está elaborado teniendo en cuenta los principios de la metodología AICLE (CLIL) que ofrece la posibilidad de aprender los contenidos curriculares de la asignatura de Matemáticas a la vez que permite practicar la lengua inglesa aprendida en etapas previas incrementando su bagaje de vocabulario técnico específico de Matemáticas en Inglés.

El hecho de que esté íntegramente elaborado en inglés constituye una ventaja para la inmersión total del alumnado en la lengua inglesa.

El alumno asimila palabras, frases y vocabulario cotidiano de la lengua inglesa además de estructuras y vocabulario específico de la propia asignatura de Matemáticas.

Es más, como es el propio alumno el que completa la tabla fonética de cada unidad, va interiorizándola sin apenas darse cuenta; tabla que puede ampliar con los términos que cada uno decida individualmente. Se trata en definitiva de un material que el alumno mismo ayuda a elaborar según sus necesidades particulares y que al final del año escolar le habrá servido para confeccionar su propia lista de vocabulario específico.

Otra ventaja de estos materiales es que por la **sencillez y concreción** con que están definidos los conceptos en el libro, se facilita el aprendizaje y se produce un impacto en la conceptualización, es decir, el alumno llega a ser capaz de pensar directamente en lo que se dice aunque esté expresado en otra lengua centrándose en los contenidos curriculares de la materia. Este aspecto ayuda a ampliar su mapa conceptual del pensamiento y a desarrollar en mayor medida sus competencias.

Y si alguna cosa más hubiera que destacar, personalmente destacaría la **motivación** que se logra en el alumnado al trabajar con este material. Por su estructura práctica, el colorido elegido al detalle para hacerlo atractivo, las listas de recursos web que se facilitan, porque las actividades y problemas planteados se ajustan a los contenidos y al nivel con propiedad, o por todo ello unido, se produce un efecto participativo y motivador que de otra forma es difícil alcanzar en la actualidad.

Por último decir que este libro está escrito con la ilusión de compartir y transmitir dos de mis grandes pasiones, la lengua inglesa y las Matemáticas, de la manera más sencilla posible.

ROSARIO CARRASCO TORRES

CONTENTS

REMEMBER THAT:

IN DECIMAL NUMBERS:

WE USE A **COMMA**

ENGLISH PEOPLE USE A **POINT**

Saying numbers:

*The figure 0 is usually called **nought** before a comma and **oh** after the comma.*

Examples:

0,60024 :nought comma six oh oh two four

***and** is used after any hundred digit and also between the whole part and the decimal part.*

Examples:

315 Three hundred and fifteen

803 Eight hundred and three

35,246 Two options:

a) Thirty-five ones and two hundred and forty-six thousandths.

b) Thirty-five comma two four six.

	CARDINALS	ORDINALS
1	One	First (1^{st})
2	Two	Second (2^{nd})
3	Three	Third (3^{rd})
4	Four	Fourth (4^{th})
5	Five	Fifth (5^{th})
6	Six	Sixth (6^{th})
7	Seven	Seventh (7^{th})
8	Eight	Eighth (8^{th})
9	Nine	Ninth (9^{th})
10	Ten	Tenth (10^{th})
11	Eleven	Eleventh (11^{th})
12	Twelve	Twelfth (12^{th})
13	Thirteen	Thirteenth (13^{th})
14	Fourteen	Fourteenth (14^{th})
15	Fifteen	Fifteenth (15^{th})
16	Sixteen	Sixteenth (16^{th})
17	Seventeen	Seventeenth (17^{th})
18	Eighteen	Eighteenth (18^{th})
19	Nineteen	Nineteenth (19^{th})
20	Twenty	Twentieth (20^{th})
21	Twenty-one	Twenty-first (21^{st})
22	Twenty-two	Twenty-second (22^{nd})
23	Twenty-three	Twenty-third (23^{rd})
24	Twenty-four	Twenty-fourth (24^{th})
25	Twenty-five	Twenty-fifth (25^{th})
26	Twenty-six	Twenty-sixth (26^{th})
27	Twenty-seven	Twenty-seventh (27^{th})
28	Twenty-eight	Twenty-eighth (28^{th})
29	Twenty-nine	Twenty-ninth (29^{th})
30	Thirty	Thirtieth (30^{th})
40	Forty	Fortieth (40^{th})
50	Fifty	Fiftieth (50^{th})
60	Sixty	Sixtieth (60^{th})
70	Seventy	Seventieth (70^{th})
80	Eighty	Eightieth (80^{th})
90	Ninety	Ninetieth (90^{th})
100	One hundred	Hundredth
1000	One thousand	Thousandth
100000	One hundred thousand	Hundred thousandth
1000000	One million	Millionth

UNIT 1: "NATURAL NUMBERS"

1. Natural numbers. Origin and Numeral systems
   - 1.1. Egyptian numeral system
   - 1.2. Mayan numeral system
   - 1.3. Roman numeral system
   - 1.4. Decimal numeral system
2. Representation and Order of the Natural numbers
3. Operations with Natural numbers
   - 3.1. Addition and subtraction
   - 3.2. Multiplication and division
   - 3.3. Powers and properties.
   - 3.4. Square roots
4. Hierarchy of the operations (Bidmas)
5. Estimating Natural numbers
   - 5.1. Rounding
   - 5.2. Truncating

KEY VOCABULARY:

To count
Natural number
Numeral system
Set
Addition
Subtraction
Digit
Order
Number line
Bigger
Smaller than
Greater than
Addend
Plus
To Sum
Minuend
Subtrahend
Minus
Difference
Commutative
Associative
Multiplication
Multiplicand
Multiplier
Distributive
Parenthesis / parentheses (pl)
Bracket
Square bracket
Division
Quotient
Dividend
Divisor
Remainder
Exact
Inexact division
Power
Base
Exponent
Index
Squared
Cubed
To raise
Square root
Radical
Radicand
Indices
Perfect square
Hierarchy
To round
To truncate

In this unit you will learn how to:

- *Express numbers using several numeral systems*
- *Operate with Natural numbers*
- *Decide correct order of the operations*
- *Estimate Natural numbers*

1. Natural numbers. Origin and Numeral systems

The **Natural numbers** are the counting numbers. The set of the Natural numbers is represented by $\mathbb{N}$.

$$\mathbb{N} = \left\{ 1, 2, 3, 4, 5, 6... \right\}$$

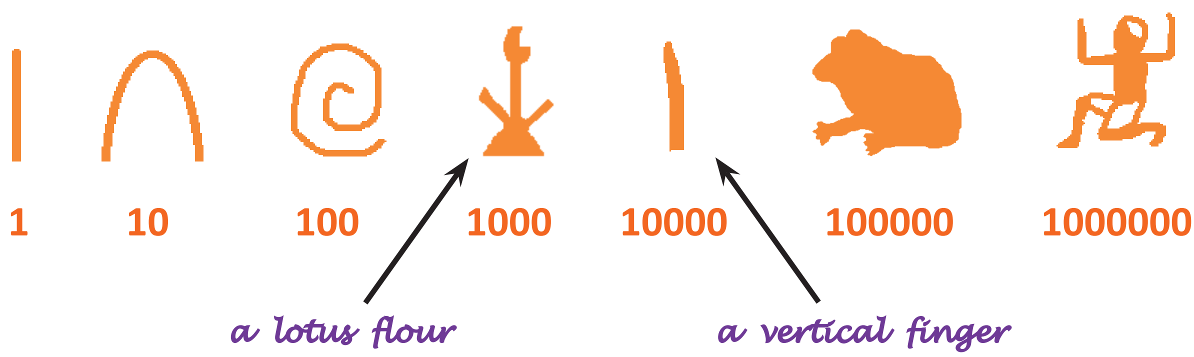
It is a NEVER ENDING SET

From the very beginning, man had the necessity for counting. Pieces of wood, stones or seeds were used to count before drawing symbols on a piece of paper.

Many civilisations have created numeral systems before the decimal system that we use nowadays. Let's see some of them:

1.1. Egyptian numeral system

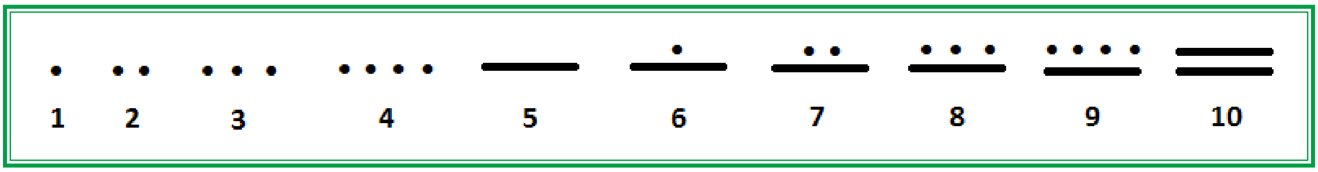
Over 3000 years ago, the ancient Egyptians created a decimal numeral system (the base was 10) formed by special symbols known as pictograms and they usually wrote them from right to left.
The main symbols and the bases of their system were:



The rest of the numbers were combinations and additions of those above.

1.2. Mayan numeral system

The Mayan civilisation created a vigesimal system (the base was 20). They expressed each number as a combination of two symbols: a dot (•) to express the units (one to four) and a dash (▬) to represent five. The Mayan wrote numbers vertically with the lowest digit on top.



1.3. Roman numeral system

The Roman created a numeral system based on letters from their alphabet. The base of the Roman system was five as a reference to the five fingers we have in each hand. The letters they used were:

It was an adding system and at the beginning it allowed any number of equal symbols but finally the rules determined that the maximum number of equal letters written together were three. Nowadays, we still use this numeral system for many purposes, such as denoting certain dates, on many clocks...

- <u>One of the most important rules of this system</u> was that when you write two letters together if the one written first place has less value than the second one, the value of the first letter must be subtracted from the value of the second one.
  This rule can ONLY be applied in the following cases:

I before V or X; X before L or C; and C before D or M

Examples: "Express the following numbers in the Roman numeral system":
4, 9, 19, 24, 60, 347, 2.153, 10.087, 42.003, 1.000.000, 3.000.000 and 4.000.000".

4 ⟶ IV ; 9 ⟶ IX ; 19 ⟶ XIX ; 24 ⟶ XXIV ; 60 ⟶ LX

347 ⟶ CCCXLVII ; 2153 ⟶ MMCLIII ; 10087 ⟶ $\overline{\text{X}}\text{LXXXVII}$

42053 ⟶ $\overline{\text{XLII}}\text{LIII}$; 1.000.000 ⟶ $\overline{\text{M}}$; 3.000.000 ⟶ $\overline{\text{MMM}}$

4.000.000 ⟶ $\overline{\overline{\overline{\text{IV}}}}$

ONE BAR on top of a letter multiplies by 1.000 the correspondent number and TWO BARS indicates millions.

1.4. Decimal numeral system

Several numeral systems have been developed over the course of history. Eventually we arrived at the **Decimal system** (its base is 10) which we use today. This system is a positional system based on ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. It means that the position of each digit represents its value as the following example shows.

Example 1: "How do you write the number 6831742?".

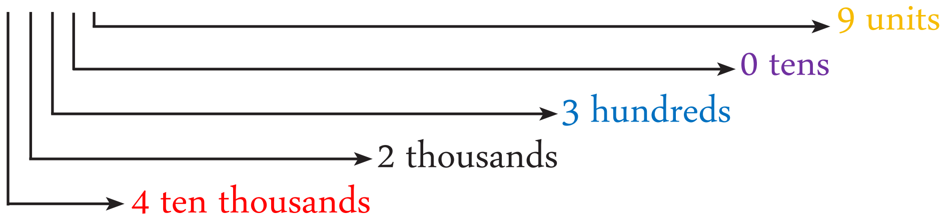
Look at the following diagram:

6	8	3	1	7	4	2
Million	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units

Therefore the number is: six million, eight hundred and thirty-one thousand, seven hundred and forty-two.

Example 2: "Determine the value of each digit in the number 42309".

42309



2. Representation and Order of the Natural numbers

- The Natural numbers can be **represented by** a line like that:

Numbers are bigger when you move to the right on the number line.

- We can use symbols **<** and **>** to express which is the **order relation** between two numbers: **less than** and **more than**.

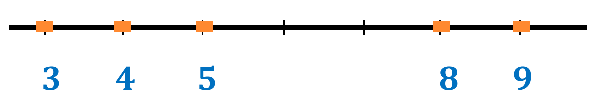
Example: "Order the following pairs of numbers: 3 and 9; 4 and 5; 9 and 8".

We can write: $3 < 9$ $4 < 5$ $9 > 8$

"3 is **smaller than** 9" "9 is **greater than** 8"

One tip:

If you have any doubts then check **the correspondent position in the number line**:



3. Operations with Natural numbers

Many situations in real life involve operations with numbers so it is useful to practise them. We strongly recommend you practise them mentally.

3.1. Addition and subtraction

- The **addition** is an operation that combines numbers to get a total. The symbol is **+**

Example: $37 + 45 = 72$

addends: 37, 45; plus: +; sum: 72

Remember the properties of the addition:

- Commutative: $a + b = b + a$ Example: $34 + 12 = 12 + 34$
- Associative: $(a + b) + c = a + (b + c)$ Example: $(2+3) + 5 = 2 + (3+5)$

Remember: solve the parenthesis first

Example: "Check the Associative property in the example above".

$$(2+3) + 5 = 2 + (3+5)$$
$$5 + 5 = 2 + 8$$
$$10 = 10$$

⇒ Then the property is correct

- The **subtraction** is an operation that takes one number away from another to get the difference. The symbol is **−** (minus)

Example: $42 - 17 = 25$

minuend: 42; subtrahend: 17; difference: 25

The subtraction is NOT Commutative

Example: $34 - 12 \neq 12 - 34$

3.2. Multiplication and division

➤ **Multiplying** natural numbers is repeating additions. The symbol is $\cdot$

You read five **multiplied by** four

Example: $5 \cdot 4 = 5 + 5 + 5 + 5 = 20$

factors

product

Remember the properties of the multiplication::

- Commutative: $a \cdot b = b \cdot a$ Example: $4 \cdot 12 = 12 \cdot 4$
- Associative: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ Example: $(2 \cdot 3) \cdot 5 = 2 \cdot (3 \cdot 5)$
- Distributive: $a \cdot (b + c) = a \cdot b + a \cdot c$ Example: $2 \cdot (6 + 3) = 2 \cdot 6 + 2 \cdot 3$
  $a \cdot (b - c) = a \cdot b - a \cdot c$ $2 \cdot (6 - 3) = 2 \cdot 6 - 2 \cdot 3$

Example 1: "Express the following calculation as a product: 15+15+15+15 +15+15 and solve it".

$$15 + 15 + 15 + 15 + 15 + 15 = 15 \cdot 6 = 90$$

Example 2: "Apply the distributive property and solve: a) $6 \cdot (3 + 7)$ b) $8 \cdot (10 - 4)$

a) $6 \cdot (3 + 7) = 6 \cdot 3 + 6 \cdot 7 = 18 + 42 = 60$

b) $8 \cdot (10 - 4) = 8 \cdot 10 - 8 \cdot 4 = 80 - 32 = 48$

Example 3: "Apply the distributive property and solve: a) $7 \cdot 2 + 7 \cdot 5 =$ b) $9 \cdot 7 - 9 \cdot 5 =$

a) $7 \cdot 2 + 7 \cdot 5 = 7 \cdot (2+5) = 7 \cdot 7 = 49$ b) $9 \cdot 7 - 9 \cdot 5 = 9 \cdot (7 - 5) = 9 \cdot 2 = 18$

➤ **Dividing** natural numbers is determining how many times one quantity is contained in another. We use the division to make groups or to share something. The symbol is **:**

Example: $15 : 3 = 5$

dividend

divisor

quotient

You read fifteen **divided by** three

YOU NEVER CAN DIVIDE BY ZERO

There are two kinds of divisions:

EXACT DIVISION

255	3
15	85
0	

INEXACT DIVISION

286	3
16	95
1	

remainder

REMEMBER THE PROOF OF DIVISIONS:

DIVIDEND = DIVISOR · QUOTIENT + REMAINDER

Example 4: "Imagine you want to pack your books in boxes. If you have ten boxes and you can put thirty books inside each of them, how many books can you pack?".

Obviously you need to do the following operation: $10 \cdot 30 = 300$
Then you can pack 300 books.

Example 5: "Today is your birthday and you want to give sweets to your class mates. You have 65 sweets and there are 21 students in your class including yourself. How many sweets are you going to give to each of your mates? Are there any sweets left for your teacher?".

65	21
2	3

For your teacher.

3.3. Powers and properties

A **power** of a number shows how many times the number is multiplied by itself.

You read a to the power of n

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$$

base

Exponent (index or power)

SPECIAL CASES:

3^2: you read it "three squared" 6^3: you read it "six cubed"

Example 1: "Raise the following numbers to the power of four and work out the powers: 3, 2, 4, 11 and 10".

$3^4 = 81$; $2^4 = 16$; $4^4 = 256$; $11^4 = 14641$; $10^4 = 10000$

Notice the number of zeros and the index.

Example 2: "Express the following multiplications as powers:

a) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 =$ b) $7 \cdot 7 \cdot 7 \cdot 7 =$ c) $6 \cdot 6 =$ ".

The powers are: a) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5$ b) $7 \cdot 7 \cdot 7 \cdot 7 = 7^4$ c) $6 \cdot 6 = 6^2$

Example 3: "Calculate: a) Seven to the power of three b) Six to the fourth power c) Eight squared d) Two cubed e) Nine to the fifth".

The results are: a) $7^3 = 343$ b) $6^4 = 1296$ c) $8^2 = 64$ d) $2^3 = 8$ e) $9^5 = 58049$

Properties of the powers:

- The product of powers with the same base is another power with the same base and the exponent is the addition of the exponents.

  $a^n \cdot a^m = a^{n+m}$ Example: $2^5 \cdot 2^4 = 2^9$

- The quotient of powers with the same base is another power with the same base and the exponent is the subtraction of the exponents.

  $a^n : a^m = a^{n-m}$ Example: $2^7 : 2^2 = 2^5$

- The power of a power is another power with the same base and the exponent is the product of the exponents.

  $(a^n)^m = a^{n \cdot m}$ Example: $(2^3)^4 = 2^{12}$

- The power of a product (quotient) is the product (quotient) of the powers.

  $(a \cdot b)^n = a^n \cdot b^n$ Example: $(6 \cdot 3)^7 = 6^7 \cdot 3^7$

  $(a : b)^n = a^n : b^n$ Example: $(6 : 3)^7 = 6^7 : 3^7$

- A power whose exponent is zero is equal to one.

  $a^0 = 1$

- A power whose exponent one is equal to the base number.

  $a^1 = a$

You can use the properties in both directions

Example 1: "Express as a power:

a) $3^5 \cdot 3^9 = 3^{14}$ b) $5^8 : 5^4 = 5^4$ c) $7^2 \cdot 7^4 \cdot 7 \cdot 7^8 = 7^{15}$ d) $6^{14} : 6^{14} = 6^0$ ".

Example 2: "Complete: a) $3^5 \cdot 3^{\square} = 3^{14}$ b) $2^2 \cdot 2 \cdot 2^{\square} \cdot 2^8 = 2^{15}$ c) $\square^7 : \square^4 = 2^{\square}$

a) $3^5 \cdot 3^9 = 3^{14}$ b) $2^2 \cdot 2 \cdot 2^4 \cdot 2^8 = 2^{15}$ c) $2^7 : 2^4 = 2^3$

Example 3: "Calculate the squares of a chessboard. Express it as a power of two. (Remember there are 8 rows with 8 squares each.)".

The total of the squares is: $8 \cdot 8 = 2^3 \cdot 2^3 = 2^6$

Example 4: "Express as a power": a)$(3^2)^5 \cdot (3^9)^3 = 3^{37}$ b)$(5^8)^3 : 5^0 = 5^{24}$

c)$(7^2)^0 \cdot (7^3)^4 : 7^8 = 7^4$ d)$8^2 \cdot 2^4 = (2^3)^2 \cdot 2^4 = 2^{10}$

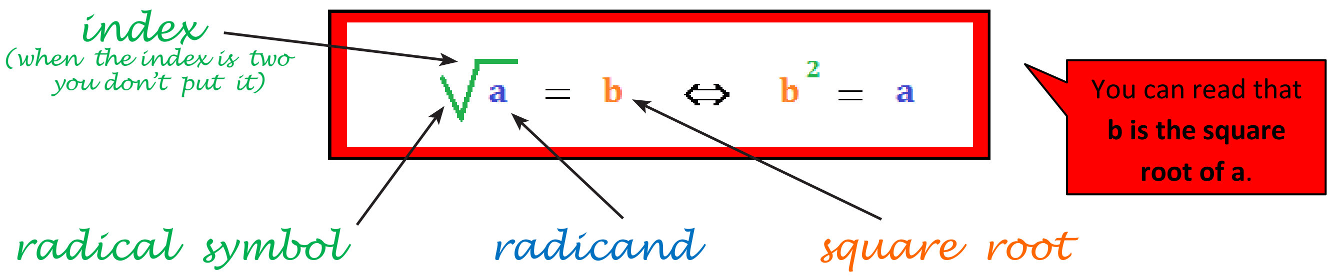
Example 5: "Calculate the value of **x** in the following powers:

a) $(9^3 : 9) = 9^x$ b) $(8^3)^x = 8^{12}$ c) $(6^x)^x = 6^{25}$ d) $(5^x)^2 : 5 = 5^5$

a) x = 2 b) x = 4 c) x = 5 d) x = 3

3.4. Square roots

The **square root** of a number **a** is another number **b** that multiplies by itself to give **a**.



Examples: "Find out the following square roots: $\sqrt{36}$, $\sqrt{9}$, $\sqrt{100}$ and $\sqrt{81}$".

$\sqrt{36} = 6$ because $6^2 = 36$ $\sqrt{9} = 3$ because $3^2 = 9$

$\sqrt{100} = 10$ because $10^2 = 100$ $\sqrt{81} = 9$ because $9^2 = 81$

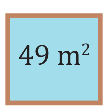
- A **perfect square** is a number that is the square of another natural number.

Examples: "Construct the list of the first eight perfect square numbers".

They are: 1, 4, 9, 16, 25, 36, 49 and 64

because $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$ and $8^2 = 64$

Example of word problem: "Imagine you know that the area of a square shaped swimming-pool is 49 m². Could you calculate the length of one of its sides?"
Yes, because you only have to calculate the square root of 49 which is 7. Then the side measures 7m.



Example: "Use your calculator to find out the square roots: $\sqrt{1296}$, $\sqrt{2025}$ and $\sqrt{10000}$".

$\sqrt{1296} = 36$ $\sqrt{2025} = 45$ and $\sqrt{10000} = 100$

Try to guess a rule for similar cases.

- **Estimating square roots:**

Sometimes the square root of a number is not a natural number. In those cases we can estimate the value of a non exact square root giving the natural number which is smaller but nearer that number.

For instance: $\sqrt{12}$ is not a whole number, nevertheless, we can estimate its value.
As we can see $\sqrt{9} < \sqrt{12} < \sqrt{16} \Rightarrow 3 < \sqrt{12} < 4 \Rightarrow \sqrt{12} \simeq 3$

Example: "Estimate the following square root: $\sqrt{18}$".

As $\sqrt{16} < \sqrt{18} < \sqrt{25} \Rightarrow 4 < \sqrt{18} < 5 \Rightarrow \sqrt{18} \simeq 4$

4. Hierarchy of the operations (Bidmas)

The hierarchy of the operations indicates the order in which they must be done correctly. The correct order is usually remembered thanks to a mnemonic rule called:

BIDMAS

Brackets – Indices – Divisions – Multiplications – Additions – Subtractions

Example: "Solve the following calculations in the correct order".

a) $38 + \underbrace{3 \cdot 4} - \underbrace{5 \cdot 6} = 38 + 12 - 30 = 20$ b) $\underbrace{(\underbrace{14 - 6}) \cdot 2} - \underbrace{2 \cdot 3} = \underbrace{8 \cdot 2} - 6 = 16 - 6 = 10$

c) $\underbrace{3 \cdot 5^2} + \sqrt{16} - \underbrace{2^3 : 4} = 3 \cdot 25 + 4 - \underbrace{8 : 4} = 75 + 4 - 2 = 77$

d) $2 + \underbrace{\sqrt{36} : 3} \cdot \underbrace{(\underbrace{4^3 - 3^2})} - (\underbrace{\sqrt{81} - 5}) = 2 + 6 : 3 \cdot (64 - 9) - (9 - 5) = 2 + \underbrace{2 \cdot 55} - 4 =$

$= 2 + 110 - 4 = 108$

Solve them from right to left

e) $\underbrace{[\underbrace{(24 - \underbrace{3 \cdot 4})} : 2^2]} \cdot 9 - \underbrace{2 \cdot 5} = [(24-12) : 4] \cdot 9 - 10 = [12 : 4] \cdot 9 - 10 = 3 \cdot 9 - 10 = 17$

Solve the brackets from the inside out

[ ] this symbol is called **square brackets**

5. Estimating Natural numbers

Estimating a Natural number is substituting it by another natural number which is close to that number. There are many ways of estimating numbers. Rounding and Truncating are two of the ways of estimating numbers.

5.1. Rounding

To **Round** a Natural number to a specific place value we look at the next digit to the right and:

a) if the digit is **5 or more** ⇒ **add one unit** to the last digit and substitute the rest of the digits by zeros.

For instance: Round off 42752 to the nearest hundred ⇒ 42800

b) if the digit is **4 or less** ⇒ simply **write** the number and add the correspondent zeros.

For instance: Round off 71423 to the nearest thousand ⇒ 71000

Example 1: "Round off 239678 to the nearest ten thousand".

239678 ⟹ 240000

Example 2: "Round off 86321 to the nearest hundred".

86321 ⟹ 86300

5.2. Truncating

To **Truncate** a number to a specific place value we simply write the number to the specified place value and drop all the remaining digits and substitute them by zeros.

For instance: "Truncate 526 to the tens" ⇒ 520

Example 1: "Truncate 140356 to the thousands".

140356 ⟹ 140000

Example 2: "Truncate 8179 to the tens".

8179 ⟹ 8170

PRACTISING NATURAL NUMBERS USING WEBSITES

⇒ http://www.ixl.com/math/grade-6/place-values-in-whole-numbers
⇒ http://www.ixl.com/math/grade-6/word-names-for-numbers
⇒ http://onlinemschool.com/math/practice/arithmetic/addition/
⇒ http://www.ixl.com/math/grade-6/roman-numerals
⇒ http://www.mathsisfun.com/definitions/power.html

VOCABULARY	*phonetics*	meaning
To count		
Natural number		
Numeral system		
Set		
Addition		
Subtraction		
Digit		
Order		
Number line		
Bigger / Greater than		
Smaller than		
Addend		
Plus		
To Sum		
Minuend		
Subtrahend		
Minus		
Difference		
Commutative		
Associative		
Multiplication		
Multiplicand / Multiplier		
Distributive		
Parenthesis / parentheses (pl)		
Bracket / Square bracket		
Division / Quotient		
Dividend		
Divisor		
Remainder		
Exact / Inexact division		
Power		
Base		
Exponent / Index		
Squared / Cubed		
To Raise		
Square root		
Radical		
Radicand		
Perfect square		
Hierarchy		
To round / to truncate		

EXERCISES

1. Express the following numbers in Roman numerals:
   a) 45 b) 125 c) 709 d) 1024 e) 3487
   f) 12300 g) 4890104 h) 27892 i) 111
   j) 999 k) 84120 l) 170025 m) 1000005
2. Write the value of the following Roman numbers in the decimal system:
   a) DLII b) XXXIX c) CCIII d) LVI e) $\overline{\mathrm{M}}\mathrm{CCCXX}$ f) $\overline{\mathrm{X}}\mathrm{XVII}$ g) MMXXII
   h) $\overline{\mathrm{CD}}\mathrm{LXXIV}$ i) $\overline{\mathrm{MM}}\mathrm{DXL}$ j) $\overline{\mathrm{IV}}\mathrm{CLXXII}$ k) $\overline{\mathrm{M}}\mathrm{MMCDLXXXIII}$ l) $\overline{\mathrm{CM}}\mathrm{XXX}$
3. Determine the value of the digit 4 in each number:
   a) 45678 b) 140 c) 4000001 d) 3142 e) 73456 f) 704
4. Write three natural numbers between two thousand and two thousand and four hundred that have 3 as the tens digit.
5. Write five natural numbers smaller than one hundred and greater than fifty and order them using the correct symbols.
6. Order the following natural numbers using the correct symbols: 3, 8, 4, 12, 15, 11, 121, 212, 33, 43.
7. Fill in the boxes:
   a) 4 < ☐ <6 b) 4 > ☐ >2 c) 8 ☐ 9 < 12
   d) 3 > ☐ >1 e) 7 ☐ 8 ☐ 9 f) ☐ >6> ☐
8. Express as a product and solve:
   a) 11 + 11+ 11+ 11= b) 31 + 31 + 31= c) 6 + 6 + 6 + 6 + 6 + 6=
   d) 7 + 7 + 7 + 7 + 7=
9. Apply the distributive property and solve:
   a) 2·(4 + 5)= b) 6·(7 – 5)= c) 4·(9 – 2)= d) 8·(5 + 6)=
   e) 7·(10 – 8)= f) 5·(11 + 12)=
10. Apply the distributive property and solve them (notice that in this case the expressions are written in the opposite direction than in the previous exercise, it also can be asked as **"Take out the common factor"**):
    a) 23 · 8 + 5 · 8 = b) 6 · 7 – 5 · 7 + 7 ·2 =
    c) 12 · 9 + 5 · 9 = d) 9 · 7 – 9 · 2 + 9 · 3 =
11. Solve the following divisions and check them using the proof of divisions:
    a) 5678 : 24 b) 34527 : 15 c) 6789 : 78
    d) 98002 : 63 e) 123456: 321
12. Calculate the **unknown** in each case:
    a) $\begin{array}{r|l} 345 & \underline{\mathbf{d}} \\ 20 & 13 \end{array}$ b) $\begin{array}{r|l} 205 & \underline{13} \\ \mathbf{r} & 15 \end{array}$ c) $\begin{array}{r|l} 1502 & \underline{27} \\ 17 & \mathbf{Q} \end{array}$ d) $\begin{array}{r|l} \mathbf{D} & \underline{23} \\ 8 & 654 \end{array}$
13. Express as a power and calculate:
    a) 3 · 3 · 3 · 3 = b) 7 · 7 · 7 = c) 5 · 5 · 5 · 5 · 5 =
    d) 11 · 11 = e) 17 · 17 · 17 =

14. Fill in the boxes applying the properties of powers:
   a) $5^3 \cdot 5^2 \cdot 5 = 5^{\square}$ b) $2^2 \cdot 2 \cdot 2^5 = 2^{\square}$ c) $\square^3 \cdot 7^{\square} = 7^8$ d) $3^9 : 3^{\square} = 3^7$

15. Work out the following powers using the right properties:
   a) $7^0 =$ b) $(2^3)^4 =$ c) $(7^4)^2 =$
   d) $5^3 : 5^3 =$ e) $(11^2)^3 : 11^5 =$ f) $13^5 : 13^2 \cdot 13 =$

16. Write as a power of a power: a) 2^6 b) 5^8 c) 7^{12} d) 9^{10} e) 13^{14} f) 25^{20}

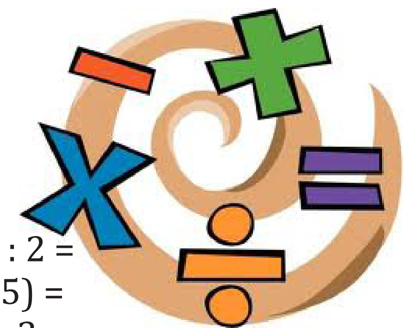
17. Express as a quotient of powers: a) 3^2 b) 6^{18} c) 72^7 d) 19^{12} e) 15^4 f) 13^5

18. Calculate the value of the unknowns in the following powers:
   a) $(3^6 : 3) = 3^x$ b) $(8^7) : a^2 = 8^x$ c) $(7^x)^x = b^{49}$ d) $(8^x)^6 : 8 = c^{35}$

19. Fill in the boxes:
   a) $\sqrt{\square} = 9$ b) $(\sqrt{49})^2 = \square$ c) $\sqrt{\square} = 11$ d) $\sqrt{\square} \cdot \sqrt{\square} = 9$ e) $(\sqrt{\square})^2 = 5$

20. Estimate the value the following square roots:
   a) $\sqrt{11}$ b) $\sqrt{5}$ c) $\sqrt{69}$ d) $\sqrt{20}$ e) $\sqrt{51}$ f) $\sqrt{89}$

21. Solve the following operations in the correct order:



   a) $3 \cdot (5 - 2 + 4) =$ b) $7 + 8 \cdot (11 - 2 - 4) =$
   c) $71 - 7 \cdot 2 - 5 \cdot 4 =$ d) $23 + 5 \cdot 6 - 12 =$
   e) $13 - (8 : 4) \cdot 2 =$ f) $242 : (10 : 5) =$
   g) $12 - 2 \cdot 3 - 14 : 7 =$ h) $(23 - 4 \cdot 5 - 1) : 2 =$
   i) $(36 : 4) \cdot 2 - 5 =$ j) $14 - 8 : 2 - 27 : 9 + 4 : 2 =$
   k) $34 - (3 + (15 + 7)) =$ l) $21 \cdot 3 - 12 : 2 - (40 : 5) =$
   m) $(180 - 87 : 3) - (9 \cdot 4) =$ n) $8 + 7 \cdot (15 - 7) - 24 : 3 =$
   o) $(14 - 2 + 7 + 6) : (17 - 12) =$ p) $45 - (30 : 5 + 6) : 12 =$
   q) $7 \cdot 15 : 3 - 3 \cdot 9 =$ r) $[350 - 4 \cdot (12 - 7) \cdot 2] - 7 \cdot 8 - 9 : 3 =$
   s) $(5 \cdot 5 - 3 \cdot 3 + 4 \cdot 4) - (3 \cdot 3 \cdot 3 - 2 \cdot 2 \cdot 2) =$

22. Round the following numbers to the specified place value:
   a) 1234 to the nearest ten b) 4567 to the nearest hundred
   c) 50321 to the nearest ten thousand d) 4002 to the nearest hundred
   e) 123 to the nearest ten f) 10789 to the nearest thousand

23. Truncate the following numbers to the specified place value and compare the results with the previous exercise:
   a) 1234 to the tens b) 4567 to the hundreds
   c) 50321 to the thousands d) 4002 to the hundreds
   e) 123 to the tens f) 10789 to the thousands

24. Work out the following combined operations:
   a) $3^2 \cdot 2^3 : 2 =$
   b) $\sqrt{36} + 45 : 3^2 =$
   c) $4 \cdot \sqrt{25} - 2^4 =$
   d) $7 \cdot 69 : \sqrt{9} =$
   e) $(\sqrt{81} \cdot 3) : 9 =$
   f) $\sqrt{121} \cdot (10^2 : 5) : (2 \cdot 5) =$
   g) $[2 \cdot \sqrt{100} - (18 : 3)] + 2 \cdot [3 \cdot (2^2)^2] =$

WORD PROBLEMS

25. A number **n** is greater than 24, smaller than 35 and its units digit is 0. What number is it?
26. Write your year of birth in Roman numerals.
27. Think about a palindromic (you can read the same number in both directions) number with four digits and whose ten digit is greater than five and smaller than seven.
28. Albert has scored four goals more than Peter this month, Charlie has scored 19 goals which are three less than the total of goals scored by Albert. How many goals have been scored all together?
29. John made fifty loaves of bread two weeks ago. Last week he made double this amount. How many loaves of bread has he made during the last two weeks?
30. Jane is twenty-four years old and she is three years older than her sister Anne. If you add their ages you only need to add five more to obtain the age of their mother. How old are Anne and her mother?
31. A train travels at 245 km/h, another train travels at 180km/h.

    What is the difference in km between both of the trains in 6 hours?
32. Kate has a date. She wants to invite her boyfriend and she also wants to buy a dress and some accessories for that date. The price of a meal in a restaurant is between 20 and 30 euros per person. The price of a dress in a shop is between 80 and 200€ euros. The price of the accessories including shoes is between 55 and 92 euros in the same shop. What are the minimum and maximum expenses for the day?
33. A car travels 120 km in one hour. How many km does it travel a week if it spends eight hours a day travelling including the weekend?
34. Sue wants to buy a dog. It costs 450 euros. She gets 20 euros of pocket money a week and she manages to save half of it. How many weeks does she need to save enough to buy the dog?
35. Anne has bought 12 boxes and each one contains 12 bottles of wine. How many bottles of wine has she bought?
36. Each milk carton costs one euro. Today there is a special offer: if you buy three cartons you only pay two cartons. Alice needs 18 cartons for a month. How much money does she save if she decides to buy all those cartons today?
37. Kelly wins a prize in a contest. The prize is 125000 euros and she wants to give it to her three children to pay for their careers. One of them is disabled and she wants him to have double the amount of money. How much money is each of them going to receive?

38. Diana spends 35 euros to pay the electricity bill, 24 euros to pay the telephone bill and 22 euros to pay the water bill each month. How much does she need to pay all those bills in a year?
39. Imagine you want to invite your five friends to the cinema and to have popcorn as well. Each ticket costs eight euros. The box of popcorn is two, three or four euros depending on the size of the box. What is the minimum and the maximum of money you have to pay depending on the sizes of the popcorn boxes they choose? (In each case the size will be the same for all of your friends).
40. Gabrielle works in a library and she wants to know what the total of all the books there can be. There are fifty-two bookshelves with seven shelves each and 32 books by shelf. Can you help Gabrielle?
41. The owner of a restaurant wants to move it to another city. The workers have to pack all the glasses. Each box has five rows and eight glasses can be perfectly packed in each row. How many boxes do you need to pack 1520 glasses?
42. If you know that D=1425, d=36 and q=39 are elements of a division, could you say if the division is exact without doing it?
43. If you know that D=4568, d=56 and r=32 are elements of a division, calculate the quotient without doing the division.
44. A writer has written a book. The total of words is 16345. Each page contains 435 words (except the last one which has less). How many pages does the book have? How many words are in the last page?
45. Imagine you have a square shaped picture of 2025 cm^2, could you calculate the side of the picture?
46. A teacher wants to make groups of two, three or four students to do a research. There are 35 students in the class. What are the minimum and the maximum number of groups they can make?
47. Imagine you want to distribute 32 chocolates in bags of 3, 4 and 5 chocolates each. Calculate the minimum and the maximum of bags you need to distribute all the chocolates.
48. Give two numbers that rounded to the tens yield the same result.
49. Give two numbers that truncated to the hundreds yield the same result.